# Comparison of Trend Parameters and Seasonal Indices in the Presence and Absence of Missing Values of Exponential Trend-Cycle in Time Series Analysis

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

***Original Research Article***

## Abstract

This study examines the comparison of trend cycle and seasonal components in the presence and absence of missing observations. The method adopted in this study is based on the row, column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The method assumes that (1) Only one data is missing at a time in the Buys-Ballot table (2) the trending curve is exponential (3) the modal structure is additive. The study indicates that, the estimation of the missing observations as they occur consecutively with the errors being normally distributed. Results indicate that, the differences in the trend parameters for both situations are insignificant because they are approximately the same. In the case of corresponding seasonal effects, significant differences existed on the points in which there are missing values in the column of the Buys-Ballot table.

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# 1 Introduction

Missing observations in time series analysis occurred as a result of issues that have to do with technical fault and sometime in human errors. It happens when an observation may not be made at particular time because of industrial action, inability to keep proper record, faulty equipment, mistake that cannot be ratified. This factors mentioned are very common in time series analysis. The missing values may be replaced with naïve forecast or with the average of the last two known observations that bound the missing value Almed [1]. Missing observation can lead to wrong conclusions about time series data. The process of substitution of missing observations may introduce inaccuracies. It can lead to inaccurate results, forecast and errors or data skews can proliferate across subsequent runs causing a large cumulative error effect.

Brockwell and Davis [2] discussed the option that missing data at the beginning or end of the series are simply ignored while intermediate missing data are seen as problems in the input time series. Therefore, they observed that, interpolates values using interpolation algorithm linear, polynomial, smoothing, spline and filering.

Cheema [3] used different time series methods to handle missing data. This methods are mean imputation, regression imputation, maximum likelihood imputation, multiple imputation and listwise deletion. Iwueze *et.al* [4] proposed three different methods for estimation of missing values in time series. The methods are Column Mean Imputation (CMI) and Decomposing Without the Missing Value (DWMV). According to them, Decomposition Without the Missing Value (DWMV) yielded the best result when compared with the others. The reason for this study is to contribute to many existing solution of the problem of imputing missing values to a time series already in the literature.

Three time series models commonly used are additive, multiplicative and mixed models. If short period of time are involved, the trend component is superimposed into the cyclical Chatfield [5] and the observed time series $\left( X_t, t = 1, 2, ..., n \right)$ can be decomposed into the trend-cycle component $(M_t)$, seasonal component $(S_t)$ and the irregular component $(e_t)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

As far as the descriptive method of decomposition is concerned, the first step will usually be to estimate and eliminate trend-cycle ( $M_t$ ) for each time period from the actual data either by subtraction, for Equation (1) or division, for Equation (2). The de-trended series is obtained as $X_t - \hat{M}_t$ for Equation (1) or $X_t / \hat{M}_t$ for Equations (2) and (3). The seasonal effect is obtained by estimating the average of the de-trended series at each season.  The de-trended, de-seasonalized series is obtained as $X_t - \hat{M}_t - \hat{S}_t$ for Equation (1) or $X_t / \left( \hat{M}_t \hat{S}_t \right)$ for Equations (2) and (3). This gives the residual or irregular component. Having fitted a

time series model, one often wants to see if the residuals are purely random. For details of residual analysis, see Box, *et al*, [6] and Ljung and Box [7]. It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t, \text{ for all } t \tag{4}$$

For Equation (1), it is assumed to make the further assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0. \tag{5}$$

Similarly, for Equations (2) and (3), it is also assumed to make further assumption is that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s. \tag{6}$$

In this article, the use of Buys-Ballot table for estimation of trend parameters and seasonal indices in the presence of missing data using the methods of Iwueze and Nwogu [8] will be shown. The missing values will be estimated using the decomposition methods of Iwueze et.al [4] and entire process of estimation will repeated in the absence of missing data. The effect of Buys-Ballot estimation of exponential trend parameters and seasonal indices in the presence and absences of missing data will be determined.

This article is limited to time series with exponential trend that admits the additive model using registered number of reported infant baptism over the period January, 2012 to December, 2021. The observed series is transformed and the trend parameters and seasonal indices estimated using Decomposing Without the Missing Value. The missing observations will be estimated using this decomposition method and the entire process.

## 2 Materials and Methods

### 2.1 Buys-ballot procedure

The method adopted for this article is the Buys-Ballot method. For details of this method, see Iwueze et.al [4], Nwogu et.al [9], Dozie and Uwaezuoke [10], Dozie and Ibebuogu [11], Dozie et.al [12], Dozie and Ijomah [13], Dozie and Nwanya [14], Dozie [15], Dozie and Uwaezuoke [16], Dozie and Ihekuna [17], Dozie and Ibebuogu [18], Akpanta and Iwueze [19]. Dozie and Ihekuna [20], Dozie [21] and Dozie and Uwaezuoke [22].

### 2.2 Column mean imputation (CMI)

The mean imputation is given.

$$CMI = \hat{X}_{(i-1)s+j} = \frac{1}{m-1}\left[\sum_{u-1}^{i=1} X_{(u-1)s+j} + \sum_{u-i+1}^{m} X_{(u-1)s+j}\right] \tag{7}$$

### 2.3 Decomposing without missing value (DWMV)

The estimates of the missing value at $(i-1)s + j$ of the trend-cycle component of the regression imputation method for the exponential is given as

$$\hat{M}_{(i-1)s+j} = \hat{b}\, e^{\hat{c}\left[(i-1)s+j\right]} \tag{8}$$

## 2.4 Estimation of trend cycle and seasonal components for exponential

The estimates of trend cycle and seasonal components provided by Iwueze and Nwogu [8]

$$\hat{C} = \frac{C'}{S} \tag{9}$$

$$\hat{b} = b' \ell^{c\left(\frac{s-1}{2}\right)} \tag{10}$$

$$\hat{S}_{.j} = \overline{X}_{.j} - \overline{X}_{..} \tag{11}$$

$$\text{where } \overline{X}_{.j} = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)e^{cj} + s_j \tag{12}$$

$$\text{and } \overline{X}_{..} = \frac{be^c}{n}\left(\frac{1-e^{cn}}{1-e^c}\right) \tag{13}$$

## 2.5 Estimates of the missing values of transformed series

The estimates of the missing values of transformed series.

$$\hat{X}_{ij} = \hat{b}\ell^{\hat{c}\left[(i-1)s+j\right]} + \hat{S}_j + \overset{\wedge}{e}_{ij} \tag{14}$$

## 2.6 Choice of appropriate transformation

Akpanta and Iwueze [19] presented the slope of the regression equation of log of group standard deviation on log of group mean as stated in equation (15) is what is needed for choice of appropriate transformation. Some of the values of slope $\beta$ and their implied transformation are stated in Table 1.

$$\log_e \overset{\wedge}{\sigma}_i = a + \beta \log_e \overset{\wedge}{X}_{i.} \tag{15}$$

**Table 1. Bartlett's Transformation for Some Values of $\beta$**

| S/No | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | 3 | -1 |
| Transformation | No transformation | $\sqrt{X_t}$ | $\log_e X_i$ | $\frac{1}{\sqrt{X_t}}$ | $\frac{1}{X_t}$ | $\frac{1}{X_t^2}$ | $X_t^2$ |

The method of Akpanta and Iwueze [19] is applied in selecting the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the $\beta$ - value will determine the type of transformation.

# 3 Analysis

This section presents empirical example to illustrate the application of the methods of estimating missing data discussed in chapter 2. One hundred and twenty (120) of reported cases of registered infant baptism at St Jude

Church Amuzi, Mbaise in Imo State, Nigeria from January, 2012 to December, 2021 are considered in which these points have two (2) missing values. One hundred and eighteen (118) observed values are shown in Appendix A. The time plots of actual and transformed series with missing data are given in Fig. 1. The amplitude of Figs. 1and 2 appears small in the first two months and appears to have increased in the later year indicating that the variance is not constant, suggesting that the data requires transformation to stabilize the variance. The natural logarithm of the periodic and standard deviation are given in Table 4. The data is transformed by using the natural logarithm of the one hundred and eighteen (118) observations by the method of Akpanta and Iwueze [19]. The row and column totals, means and standard deviations in the presence of missing values are shown in Tables 3 and 4 respectively.

**Table 2.  Natural logarithm of periodic averages and standard deviations with missing values**

| $\overline{X}_{i.}$ | $Log_e \overline{X}_{i.}$ | $\hat{\sigma}_{i.}$ | $Log_e \hat{\sigma}_{i.}$ |
|---|---|---|---|
| 9.00 | 2.20 | 4.69 | 1.55 |
| 6.82 | 1.92 | 2.48 | 0.91 |
| 6.67 | 1.90 | 2.84 | 1.04 |
| 6.42 | 1.86 | 3.58 | 1.28 |
| 7.67 | 2.04 | 3.70 | 1.31 |
| 7.08 | 1.96 | 4.12 | 1.46 |
| 9.83 | 2.29 | 4.63 | 1.53 |
| 8.92 | 2.19 | 3.60 | 1.28 |
| 7.33 | 1.99 | 5.84 | 1.76 |
| 9.50 | 2.25 | 3.92 | 1.37 |

The estimates of missing values using Column Mean Imputation in equation (7)

The missing values are: First missing observation

$$\frac{1}{10-1}\left[4+\left(4+3+4+3+6+7+4+5\right)\right]=\frac{40}{9}=4.44 \approx 4$$

Second missing observation

$$\frac{1}{10-1}\left[12+7+10+5+5+19+13+3+12\right]=\frac{86}{9}=9.56 \approx 10$$

These are the missing values estimated using Column Mean Imputation (CMI) in equation (1), the entire process of estimation is repeated in the  absence using the method of Decomposing Without the Missing Value (DWMV) in equation (8). The results from trend parameters and seasonal indices with and without missing values are therefore compared.

## 3.1 Estimates of exponential trend-cycle and seasonal effect with missing data

Using equations (9), (10) and (11) we obtain b′ = 1.758 and c′ = 1.001

$$\hat{C}=\frac{1.001}{12}=0.0834$$

$$\hat{b}=1.758\times \ell^{0.0834\left(\frac{12-1}{2}\right)}=1.758\times \ell^{0.4587}=1.758\times 1.5820=2.781$$

$$\hat{S}_{.j}=\overline{X}_{.j}-\overline{X}_{..}$$

**Table 3. Estimates of seasonal indices with missing values**

| $j$ | $\bar{X}._{j}$ | $\hat{S}._{j}$ | $Adj\ \hat{S}_{j}$ |
|---|---|---|---|
| 1 | 1.575 | -0.271 | -0.269 |
| 2 | 1.821 | -0.025 | -0.024 |
| 3 | 1.943 | 0.097 | 0.098 |
| 4 | 1.865 | 0.019 | 0.020 |
| 5 | 1.835 | -0.011 | -0.010 |
| 6 | 1.730 | -0.116 | -0.114 |
| 7 | 1.955 | 0.109 | 0.111 |
| 8 | 1.814 | -0.032 | -0.031 |
| 9 | 1.823 | -0.023 | -0.021 |
| 10 | 1.858 | 0.012 | 0.013 |
| 11 | 2.059 | 0.213 | 0.214 |
| 12 | 1.858 | 0.012 | 0.013 |
| $\sum_{j=1}^{12} \hat{S}._{j}$ | | -0.016 | 0.000 |

The exponential trend-cycle and seasonal indices for the transformed data with missing data are estimated using Decomposition Without Missing Value (DWMV) method and the missing data are also estimated which are shown in Tables 3, 4 and 5 respectively

**Table 4. Seasonal effect with missing data**

| Parameters | With missing values |
|---|---|
| $\hat{b}$ | 1.758 |
| $\hat{c}$ | 1.001 |
| $\hat{S}_{1}$ | -0269 |
| $\hat{S}_{2}$ | -0.024 |
| $\hat{S}_{3}$ | 0.098 |
| $\hat{S}_{4}$ | 0.020 |
| $\hat{S}_{5}$ | 0.010 |
| $\hat{S}_{6}$ | -0.114 |
| $\hat{S}_{7}$ | 0.111 |
| $\hat{S}_{8}$ | -0.031 |
| $\hat{S}_{9}$ | -0.021 |
| $\hat{S}_{10}$ | 0.013 |
| $\hat{S}_{11}$ | 0.214 |
| $\hat{S}_{12}$ | 0.013 |

The estimates of missing data with the transformed data and missing position are displayed in Table 8.

**Table 5. Estimates of the Transformed and True Missing Data**

| Missing Position | $X_{2,1}$ | $X_{1,7}$ |
|---|---|---|
| Transformed Missing Data | 7.954 | 5.097 |
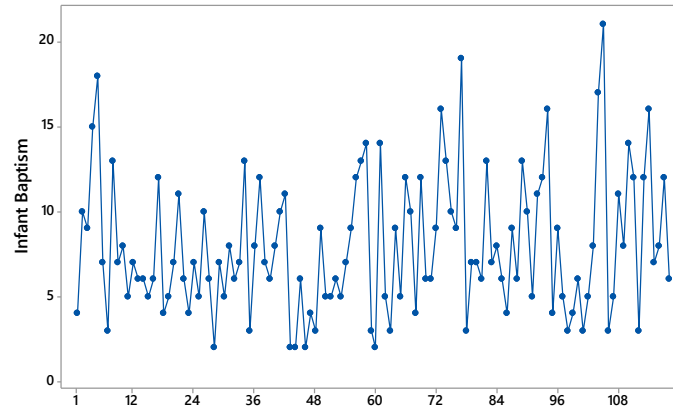


**Fig. 1. Original time series data with missing data**

## 3.2 Estimates of exponential trend-cycle and seasonal effect without missing data

Using equations (9), (10) and (11) we obtain b′ = 1.895 and c′ = 1.00

$$\hat{C} = \frac{1.00}{12} = 0.0833$$

$$\hat{b} = 1.895 \times \ell^{0.0833\left(\frac{12-1}{2}\right)} = 1.895 \times \ell^{0.4582} = 1.895 \times 1.5812 = 2.996$$

$$\hat{S}_{.j} = \overline{X}_{.j} - \overline{X}_{..}$$

**Table 6. Estimates of seasonal indices without missing values**

| $j$ | $\overline{X}_{.j}$ | $\hat{S}_{.j}$ |
|---|---|---|
| 1 | 2.213 | 0.289 |
| 2 | 1.821 | -0.103 |
| 3 | 1.943 | 0.019 |
| 4 | 1.865 | -0.059 |
| 5 | 1.835 | -0.089 |
| 6 | 1.730 | -0.194 |
| 7 | 2.269 | 0.345 |
| 8 | 1.814 | -0.110 |
| 9 | 1.823 | -0.101 |
| 10 | 1.858 | -0.066 |
| 11 | 2.059 | 0.135 |
| 12 | 1.858 | -0.066 |
| $\sum_{j=1}^{12} \hat{S}_{.j}$ | | 0.000 |

**Table 7. Estimates of exponential trend-cycle and seasonal effect without missing data**

| Parameters | Without missing values |
| --- | --- |
| $\hat{b}$ | 1.755 |
| $\hat{c}$ | 1.00 |
| $\hat{S}_1$ | 0.289 |
| $\hat{S}_2$ | -0.103 |
| $\hat{S}_3$ | 0.019 |
| $\hat{S}_4$ | 0.059 |
| $\hat{S}_5$ | -0.089 |
| $\hat{S}_6$ | -0.194 |
| $\hat{S}_7$ | 0.345 |
| $\hat{S}_8$ | -0.110 |
| $\hat{S}_9$ | -0.101 |
| $\hat{S}_{10}$ | -0.066 |
| $\hat{S}_{11}$ | 0.135 |
| $\hat{S}_{12}$ | -0.066 |
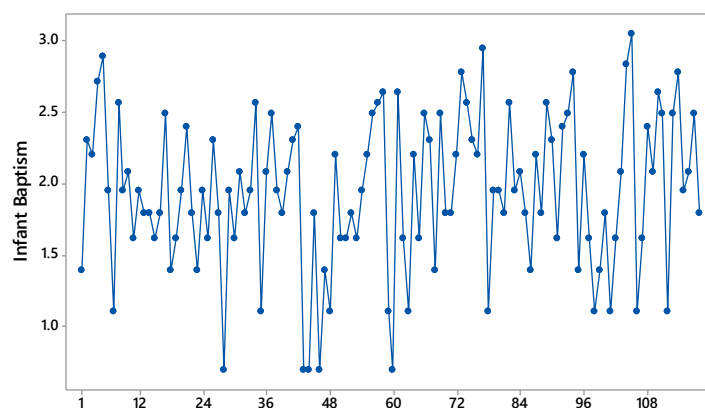


**Fig. 2. Transformed time series data of missing data**

The seasonal indices and exponential trend-cycle and seasonal indices for the transformed data without missing data are estimated using Decomposition Without Missing Value (DWMV) method are shown in Tables 6 and 7 respectively.

The row and column totals, means and standard deviations in the absence of missing values are listed in Tables 8 and 9.

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ number\ of\ observation$$

Where,

$r_i$ = Number of observation in the rth row

$c_j$ = Number of observation in the jth column.

**Table 8. Row totals, means and variances with missing observations**

| Periods $i$ | With missing observations | | | |
|---|---|---|---|---|
| | $r_i$ | $T_i$ | $\bar{X}_i$ | $\hat{\sigma}_i^2$ |
| 1 | 9 | 21.149 | 1.923 | 0.967 |
| 2 | 9 | 19.721 | 1.793 | 1.788 |
| 3 | 10 | 21.202 | 1.767 | 0.245 |
| 4 | 10 | 20.554 | 1.713 | 0.346 |
| 5 | 10 | 22.048 | 1.837 | 0.280 |
| 6 | 10 | 21.230 | 1.769 | 0.344 |
| 7 | 10 | 23.824 | 1.965 | 0.303 |
| 8 | 10 | 23.261 | 1.938 | 0.249 |
| 9 | 10 | 21.186 | 1.765 | 0.387 |
| 10 | 10 | 23.647 | 1.971 | 0.281 |
| Overall Total | 98 | 217.819 | 1.846 | 0.304 |

**Table 9. Column totals, means and variances with missing observations**

| Seasons $j$ | With missing observations | | | |
|---|---|---|---|---|
| | $c_j$ | $T_{\cdot j}$ | $\bar{X}_{\cdot j}$ | $\hat{\sigma}_{\cdot j}^2$ |
| 1 | 12 | 14.177 | 1.575 | 0.142 |
| 2 | 11 | 18.209 | 1.821 | 0.279 |
| 3 | 12 | 19.426 | 1.943 | 0.255 |
| 4 | 12 | 18.645 | 1.865 | 0.324 |
| 5 | 12 | 18.350 | 1.835 | 0.318 |
| 6 | 12 | 17.302 | 1.730 | 0.238 |
| 7 | 11 | 17.596 | 1.955 | 0.345 |
| 8 | 12 | 18.137 | 1.814 | 0.337 |
| 9 | 12 | 18.233 | 1.823 | 0.267 |
| 10 | 12 | 18.579 | 1.858 | 0.326 |
| 11 | 12 | 20.588 | 2.059 | 0.257 |
| 12 | 12 | 18.578 | 1.858 | 0.387 |
| Overall Total | 142 | 217.819 | 1.846 | 0.304 |

**Table 10. Row totals, means and variances without missing observations**

| Periods $i$ | Without missing observations | | | |
|---|---|---|---|---|
| | $r_i$ | $T_i$ | $\bar{X}_i$ | $\hat{\sigma}_i^2$ |
| 1 | 10 | 26.245 | 2.187 | 0.967 |
| 2 | 10 | 27.675 | 2.306 | 1.788 |
| 3 | 10 | 21.202 | 1.767 | 0.246 |
| 4 | 10 | 20.554 | 1.713 | 0.346 |
| 5 | 10 | 22.048 | 1.837 | 0.280 |
| 6 | 10 | 21.230 | 1.769 | 0.344 |
| 7 | 10 | 23.824 | 1.985 | 0.303 |
| 8 | 10 | 23.261 | 1.938 | 0.249 |
| 9 | 10 | 21.186 | 1.765 | 0.387 |
| 10 | 10 | 23.647 | 1.971 | 0.281 |
| Overall Total | 98 | 230.870 | 1.924 | 0.698 |

**Table 11. Column totals, means and variances without missing observations**

| Seasons $j$ | Without missing observations | | | |
|---|---|---|---|---|
| | $c_j$ | $T._j$ | $\bar{X}._j$ | $\hat{\sigma}^2._j$ |
| 1 | 12 | 22.131 | 2.213 | 2.022 |
| 2 | 12 | 18.209 | 1.821 | 0.279 |
| 3 | 12 | 19.426 | 1.943 | 0.255 |
| 4 | 12 | 18.645 | 1.865 | 0.324 |
| 5 | 12 | 18.350 | 1.835 | 0.318 |
| 6 | 12 | 17.302 | 1.730 | 0.238 |
| 7 | 12 | 22.693 | 2.269 | 1.046 |
| 8 | 12 | 18.137 | 1.814 | 0.337 |
| 9 | 12 | 18.233 | 1.823 | 0.267 |
| 10 | 12 | 18.579 | 1.858 | 0.326 |
| 11 | 12 | 20.588 | 2.059 | 0.257 |
| 12 | 12 | 18.578 | 1.858 | 0.387 |
| Overall Total | 144 | 230.870 | 1.924 | 0.698 |

**Table 12. Comparison of exponential trend-cycle and seasonal indices with and without missing data**

| Parameters | With Missing Data | Without Missing Data | Difference |
|---|---|---|---|
| $\hat{b}$ | 1.758 | 1.755 | 0.003 |
| $\hat{c}$ | 1.001 | 1.00 | 0.001 |
| $\hat{S}_1$ | -0.269 | 0.289 | 0.558 |
| $\hat{S}_2$ | -0.024 | -0.103 | 0.079 |
| $\hat{S}_3$ | 0.098 | 0.019 | 0.079 |
| $\hat{S}_4$ | 0.020 | 0.059 | 0.039 |
| $\hat{S}_5$ | 0.010 | -0.089 | 0.099 |
| $\hat{S}_6$ | -0.114 | -0.194 | 0.080 |
| $\hat{S}_7$ | 0.111 | 0.345 | 0.234 |
| $\hat{S}_8$ | -0.031 | -0.110 | 0.079 |
| $\hat{S}_9$ | -0.021 | -0.101 | 0.080 |
| $\hat{S}_{10}$ | 0.013 | -0.066 | 0.079 |
| $\hat{S}_{11}$ | 0.214 | 0.135 | 0.079 |
| $\hat{S}_{12}$ | 0.013 | -0.066 | 0.079 |

## 3.3 Comparison of the estimation of trend parameters and seasonal indices in the presence/absence of missing values

As shown in Table 12, the estimated exponential trend parameters and seasonal indices in the presence and absence of missing values are compared. The differences in the trend parameters for both situations (presence and absence) are insignificant because they are approximately the same. In the case of corresponding seasonal effects, significant differences existed on the points in which there are missing values in the column of the Buys-Ballot table.

# 4 Summary, Conclusion and Recommendations

This Study has compared the results of Buys-Ballot estimates of exponential trend cycle component and that of seasonal indices in the presence and absence of missing observations in time series. The method adopted is based on the row, Column and overall means of the time series arranged in a Buys-Ballot table with m rows and s columns. The study is limited to a series with only exponential trend and seasonal indices combined in the additive form. Only data missing at one point at a time is considered. This study provided a solution to the problem of missing observations not only one observation missing at a time but two or more observations missing consecutively.

Results indicate that, under the stated assumptions, (1) there is no significant difference in the trend parameters in the presence and absence of missing values shown in Table 12. Therefore, missing values have no effect in trend parameters. (2) there is significant effect in the seasonal indices of missing values. The differences occurred at $j = 1$ and 7 as indicted in Table 12. These are the points in the column of the Buys-Ballot table that have missing values. The estimates for the unobserved number of registered infant baptism are: (5.097) in July, 2012 and (7.954) in January, 2013. The study has provided a basis for the comparison of exponential trend cycle and seasonality in the time in the presence and absence of missing observations when only data missing at one point at a time. No attempt has been made to discuss this method when the trend-cycle component is not exponential or when the cyclical component is separated from the trend. Therefore, further investigations in these directions are recommended.

# Competing Interests

Authors have declared that no competing interests exist.

# References

[1]    Almed MR, Al-Khazaleh AMH. Estimation of missing data using the filtering process in a time series modelling; 2008.

[2]    Brockwell PJ, Davis RA. Time series theory and methods springer-verlag. New York; 1991.
       Available:https://doi.org/10.1007/978-1-4419-0320-4

[3]    Cheema IR. Some general guideline for choosing missing data handling methods in educational research. Journal of Modern Applied Statistical Methods. 2014;13:Article 3
       Available:https://doi.org/10.22237

[4]    Iwueze IS, Nwogu EC, Nlebedim VU, Nwosu UI, Chinyem UE. Comparison of methods of estimating missing values in time series. Open Journal of Statistics. 2018;8:390-399.

[5]    Chatfield C. The analysis of time Series: An introduction. Chapman and Hall,/CRC Press, Boca Raton; 2004.

[6]    Box GEP, Jenkins GM, Reinsel GC. Time series analysis: Forecasting and control (3rd ed.). Englewood Cliffs N. J, Prentice-Hall; 1994.

[7]    Ljung GM, Box GEP. On a measure of lack of fit in time series models. Biometrika. 1978;65:297-303.

[8]    Iwueze IS, Nwogu EC. Framework for choice of models and detection of seasonal effect in time series.Far East Journal of Theoretical Statistics. 2014;48(1):45– 66.

[9]    Nwogu EC, Iwueze IS. Dozie KCN, Mbachu HI. Choice between mixed and multiplicative models in time series decomposition. International Journal of Statistics and Applications. 2019;9(5):153-159.

[10]    Dozie KCN, Uwaezuoke UM. Properties of buys-ballot estimates for mixed model in time series decomposition. Galore International Journal of Applied Sciences and Humanities. 2020;4(2):35-40.

[11]    Dozie KCN, Ibebuogu CC. Estimates of time series components of road traffic accidents and effect of incomplete observations: Mixed model case. International Journal of Research and Review. 2020;7(6):343-351.

[12]    Dozie KCN, Nwogu EC, Ijeomah MA. Effect of missing observations on buys-ballot estimates of time series components. Asian Journal of Probability and Statistics. 2020;6(3):13-24.

[13]    Dozie KCN, Ijomah MA. A comparative study on additive and mixed models in descriptive time series. American Journal of Mathematical and Computer Modelling. 2020;5(1):12-17.

[14]    Dozie KCN, Nwanya JC. Comparison of mixed and multiplicative models, when trend cycle components is linear. Asian Journal of Advance Research and Reports. 2020;12(4):32-42.

[15]    Dozie KCN. Estimation of seasonal variances in descriptive time series analysis. Asian Journal of Advanced Research and Reports. 2020;10(3):37 – 47.

[16]    Dozie KCN, Uwaezuoke MU. Seasonal analysis of average monthly exchange rate of central bank of Nigeria (CBN) when trend-cycle component is quadratic. International Journal of Research and Innovation in Applied Science. 2021;6(2):201-205.

[17]    Dozie KCN, Ihekuna SO. Chi-square test in time series data. Asian Journal of Probability and Statistics. 2022;20(3):49-63.

[18]    Dozie KCN, & Ibebuogu C.C (2023). Decomposition with the mixed model in time series analysis using Buy-Ballot procedure. Asian Journal of Advanced Research and Report 17(2), 8-18

[19]    Akpanta AC, Iwueze IS. On applying the Bartlett transformation method to time series data. Journal of Mathematical Sciences. 2009;20(5):227-243.

[20]    Dozie KCN, Ihekuna SO. The effect of missing data on estimates of exponential trend-cycle and seasonal components in time series: Additive case. Asian Journal of Probability and Statistics. 2023;24(1):22-36.

[21]    Dozie KCN. Buys-ballot estimates for overall sample variances and their statistical properties: A Mixed model case. Asian Journal of Advanced Research and Reports. 2023;17(10):213-223.

[22]    Dozie KCN, Uwaezuoke MU. The proposed buys-ballot estimates for multiplicative model with the error variances. Journal of Engineering Research and Reports. 2023;25(8):94-106.

**Appendix A. Buys Ballot table of the actual data on the number of infant**

| | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{y}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 4.0 | 10.0 | 9.0 | 15.0 | 18.0 | 7.0 | | 3.0 | 13.0 | 7.0 | 8.0 | 5.0 | 99.0 | 9.00 | 4.69 |
| 2013 | | 7.0 | 6.0 | 6.0 | 5.0 | 6.0 | 12.0 | 4.0 | 5.0 | 7.0 | 11.0 | 6.0 | 75.0 | 6.82 | 2.48 |
| 2014 | 4.0 | 7.0 | 5.0 | 10.0 | 6.0 | 2.0 | 7.0 | 5.0 | 8.0 | 6.0 | 7.0 | 13.0 | 80.0 | 6.67 | 2.84 |
| 2015 | 3.0 | 8.0 | 12.0 | 7.0 | 6.0 | 8.0 | 10.0 | 11.0 | 2.0 | 2.0 | 6.0 | 2.0 | 77.0 | 6.42 | 3.58 |
| 2016 | 4.0 | 3.0 | 9.0 | 5.0 | 5.0 | 6.0 | 5.0 | 7.0 | 9.0 | 12.0 | 13.0 | 14.0 | 92.0 | 7.67 | 3.70 |
| 2017 | 3.0 | 2.0 | 14.0 | 5.0 | 3.0 | 9.0 | 5.0 | 12.0 | 10.0 | 4.0 | 12.0 | 6.0 | 85.0 | 7.08 | 4.12 |
| 2018 | 6.0 | 9.0 | 16.0 | 13.0 | 10.0 | 9.0 | 19.0 | 3.0 | 7.0 | 7.0 | 6.0 | 13.0 | 118.0 | 9.83 | 4.63 |
| 2019 | 7.0 | 8.0 | 6.0 | 4.0 | 9.0 | 6.0 | 13.0 | 10.0 | 5.0 | 11.0 | 12.0 | 16.0 | 107.0 | 8.92 | 3.60 |
| 2020 | 4.0 | 9.0 | 5.0 | 3.0 | 4.0 | 6.0 | 3.0 | 5.0 | 8.0 | 17.0 | 21.0 | 3.0 | 88.0 | 7.33 | 5.84 |
| 2021 | 5.0 | 11.0 | 8.0 | 14.0 | 12.0 | 3.0 | 12.0 | 16.0 | 7.0 | 8.0 | 12.0 | 6.0 | 114.0 | 9.50 | 3.92 |
| total | 40.0 | 74.0 | 90.0 | 82.0 | 78.0 | 62.0 | 86.0 | 76.0 | 74.0 | 81.0 | 108.0 | 84.0 | **935** | | |
| $\bar{y}_{.j}$ | 4.44 | 7.40 | 9.00 | 8.20 | 7.80 | 6.20 | 9.56 | 7.60 | 7.40 | 8.10 | 10.80 | 8.40 | | 7.92 | |
| $\sigma_{.j}$ | 1.33 | 2.88 | 3.86 | 4.44 | 4.57 | 2.30 | 5.05 | 4.43 | 3.03 | 4.28 | 4.49 | 5.06 | | | 4.07 |

**Appendix B. Buys-Ballot of transformed data without missing value**

| | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{y}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 1.537 | 2.042 | 1.976 | 2.315 | 2.450 | 1.828 | | 1.406 | 2.215 | 1.828 | 1.905 | 1.647 | 26.245 | 2.187 | 0.967 |
| 2013 | | 1.828 | 1.743 | 1.743 | 1.647 | 1.743 | 2.160 | 1.537 | 1.647 | 1.828 | 2.103 | 1.743 | 27.675 | 2.306 | 1.788 |
| 2014 | 1.537 | 1.828 | 1.647 | 2.042 | 1.743 | 1.240 | 1.828 | 1.647 | 1.905 | 1.743 | 1.828 | 2.215 | 21.202 | 1.767 | 0.246 |
| 2015 | 1.406 | 1.905 | 2.160 | 1.828 | 1.743 | 1.905 | 2.042 | 2.103 | 1.240 | 1.240 | 1.743 | 1.240 | 20.554 | 1.713 | 0.346 |
| 2016 | 1.537 | 1.406 | 1.976 | 1.647 | 1.647 | 1.743 | 1.647 | 1.828 | 1.976 | 2.160 | 2.215 | 2.266 | 22.048 | 1.837 | 0.280 |
| 2017 | 1.406 | 1.240 | 2.266 | 1.647 | 1.406 | 1.976 | 1.647 | 2.160 | 2.042 | 1.537 | 2.160 | 1.743 | 21.230 | 1.769 | 0.344 |
| 2018 | 1.743 | 1.976 | 2.362 | 2.215 | 2.042 | 1.976 | 2.491 | 1.406 | 1.828 | 1.828 | 1.743 | 2.215 | 23.824 | 1.985 | 0.303 |
| 2019 | 1.828 | 1.905 | 1.743 | 1.537 | 1.976 | 1.743 | 2.215 | 2.042 | 1.647 | 2.103 | 2.160 | 2.362 | 23.261 | 1.938 | 0.249 |
| 2020 | 1.537 | 1.976 | 1.647 | 1.406 | 1.537 | 1.743 | 1.406 | 1.647 | 1.905 | 2.407 | 2.570 | 1.406 | 21.186 | 1.765 | 0.387 |
| 2021 | 1.647 | 2.103 | 1.905 | 2.266 | 2.160 | 1.406 | 2.160 | 2.362 | 1.828 | 1.905 | 2.160 | 1.743 | 23.647 | 1.971 | 0.281 |
| total | **22.131** | **18.209** | **19.426** | **18.645** | **18.350** | **17.302** | **22.693** | **18.137** | **18.233** | **18.579** | **20.588** | **18.578** | **230.870** | | |
| $\bar{y}_{.j}$ | **2.213** | **1.821** | **1.943** | **1.865** | **1.835** | **1.730** | **2.269** | **1.814** | **1.823** | **1.858** | **2.059** | **1.858** | | **1.924** | |
| $\sigma_{.j}$ | **2.022** | **0.279** | **0.255** | **0.324** | **0.318** | **0.238** | **1.046** | **0.337** | **0.267** | **0.326** | **0.257** | **0.387** | | | 0.698 |

**Appendix C. Buys Ballot table of the actual data on the number of infant**

| | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{y}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 4.0 | 10.0 | **9.0** | **15.0** | **18.0** | **7.0** | **10.0** | **3.0** | **13.0** | **7.0** | **8.0** | **5.0** | 99.0 | 9.00 | 4.69 |
| 2013 | 4.0 | 7.0 | **6.0** | **6.0** | **5.0** | **6.0** | **12.0** | **4.0** | **5.0** | **7.0** | **11.0** | **6.0** | 75.0 | 6.82 | 2.48 |
| 2014 | 4.0 | 7.0 | **5.0** | **10.0** | **6.0** | **2.0** | **7.0** | **5.0** | **8.0** | **6.0** | **7.0** | **13.0** | 80.0 | 6.67 | 2.84 |
| 2015 | 3.0 | 8.0 | **12.0** | **7.0** | **6.0** | **8.0** | **10.0** | **11.0** | **2.0** | **2.0** | **6.0** | **2.0** | 77.0 | 6.42 | 3.58 |
| 2016 | 4.0 | 3.0 | **9.0** | **5.0** | **5.0** | **6.0** | **5.0** | **7.0** | **9.0** | **12.0** | **13.0** | **14.0** | 92.0 | 7.67 | 3.70 |
| 2017 | 3.0 | 2.0 | **14.0** | **5.0** | **3.0** | **9.0** | **5.0** | **12.0** | **10.0** | **4.0** | **12.0** | **6.0** | 85.0 | 7.08 | 4.12 |
| 2018 | 6.0 | 9.0 | **16.0** | **13.0** | **10.0** | **9.0** | **19.0** | **3.0** | **7.0** | **7.0** | **6.0** | **13.0** | 118.0 | 9.83 | 4.63 |
| 2019 | 7.0 | 8.0 | **6.0** | **4.0** | **9.0** | **6.0** | **13.0** | **10.0** | **5.0** | **11.0** | **12.0** | **16.0** | 107.0 | 8.92 | 3.60 |
| 2020 | 4.0 | 9.0 | **5.0** | **3.0** | **4.0** | **6.0** | **3.0** | **5.0** | **8.0** | **17.0** | **21.0** | **3.0** | 88.0 | 7.33 | 5.84 |
| 2021 | 5.0 | 11.0 | **8.0** | **14.0** | **12.0** | **3.0** | **12.0** | **16.0** | **7.0** | **8.0** | **12.0** | **6.0** | 114.0 | 9.50 | 3.92 |
| total | 40.0 | 74.0 | 90.0 | 82.0 | 78.0 | 62.0 | 86.0 | 76.0 | 74.0 | 81.0 | 108.0 | 84.0 | **935** | | |
| $\bar{y}_{.j}$ | 4.44 | 7.40 | 9.00 | 8.20 | 7.80 | 6.20 | 9.56 | 7.60 | 7.40 | 8.10 | 10.80 | 8.40 | | 7.92 | |
| $\sigma_{.j}$ | 1.33 | 2.88 | 3.86 | 4.44 | 4.57 | 2.30 | 5.05 | 4.43 | 3.03 | 4.28 | 4.49 | 5.06 | | | 4.07 |

**Appendix D. Buys-Ballot of transformed data without missing value**

|  | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{y}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 1.537 | 2.042 | 1.976 | 2.315 | 2.450 | 1.828 | 5.097 | 1.406 | 2.215 | 1.828 | 1.905 | 1.647 | 26.245 | 2.187 | 0.967 |
| 2013 | 7.954 | 1.828 | 1.743 | 1.743 | 1.647 | 1.743 | 2.160 | 1.537 | 1.647 | 1.828 | 2.103 | 1.743 | 27.675 | 2.306 | 1.788 |
| 2014 | 1.537 | 1.828 | 1.647 | 2.042 | 1.743 | 1.240 | 1.828 | 1.647 | 1.905 | 1.743 | 1.828 | 2.215 | 21.202 | 1.767 | 0.246 |
| 2015 | 1.406 | 1.905 | 2.160 | 1.828 | 1.743 | 1.905 | 2.042 | 2.103 | 1.240 | 1.240 | 1.743 | 1.240 | 20.554 | 1.713 | 0.346 |
| 2016 | 1.537 | 1.406 | 1.976 | 1.647 | 1.647 | 1.743 | 1.647 | 1.828 | 1.976 | 2.160 | 2.215 | 2.266 | 22.048 | 1.837 | 0.280 |
| 2017 | 1.406 | 1.240 | 2.266 | 1.647 | 1.406 | 1.976 | 1.647 | 2.160 | 2.042 | 1.537 | 2.160 | 1.743 | 21.230 | 1.769 | 0.344 |
| 2018 | 1.743 | 1.976 | 2.362 | 2.215 | 2.042 | 1.976 | 2.491 | 1.406 | 1.828 | 1.828 | 1.743 | 2.215 | 23.824 | 1.985 | 0.303 |
| 2019 | 1.828 | 1.905 | 1.743 | 1.537 | 1.976 | 1.743 | 2.215 | 2.042 | 1.647 | 2.103 | 2.160 | 2.362 | 23.261 | 1.938 | 0.249 |
| 2020 | 1.537 | 1.976 | 1.647 | 1.406 | 1.537 | 1.743 | 1.406 | 1.647 | 1.905 | 2.407 | 2.570 | 1.406 | 21.186 | 1.765 | 0.387 |
| 2021 | 1.647 | 2.103 | 1.905 | 2.266 | 2.160 | 1.406 | 2.160 | 2.362 | 1.828 | 1.905 | 2.160 | 1.743 | 23.647 | 1.971 | 0.281 |
| total | **22.131** | **18.209** | **19.426** | **18.645** | **18.350** | **17.302** | **22.693** | **18.137** | **18.233** | **18.579** | **20.588** | **18.578** | **230.870** | | |
| $\bar{y}_{.j}$ | **2.213** | **1.821** | **1.943** | **1.865** | **1.835** | **1.730** | **2.269** | **1.814** | **1.823** | **1.858** | **2.059** | **1.858** | | **1.924** | |
| $\sigma_{.j}$ | **2.022** | **0.279** | **0.255** | **0.324** | **0.318** | **0.238** | **1.046** | **0.337** | **0.267** | **0.326** | **0.257** | **0.387** | | | 0.698 |