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Some Basic Topological Properties on Non-Newtonian Real Line

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Abstract

Aims/ objectives: In this work we study some important concepts and theorems known for real line such as limit point, open and closed set and Bolzano-Weierstrass theorem in non-Newtonian real line.

Keywords:non-Newtonian real line, non-Newtonian arithmetic.

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1 Introduction

A generator is a one-to-one function whose domain is $\mathbb{R}$, the set of all real numbers, and whose range is a subset of $\mathbb{R}$. The concept of non-Newtonian calculus was firstly introduced and studied by Grossman and Katz [1]. Later, the non-Newtonian calculus is studied by Bashirov et al. [2], Ozyapici et al. [3], Filip and Piatecki [4], Çakmak and Başar [5] and others [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. We take the generator as one-to-one and also continuous function, thereafter

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we study the basic concepts and theorems over non-Newtonian real line. Identity function and exponential function can be given as examples of generators. Let the generator α be as mentioned above. We will use the same notations of reference [5].We denote by $\mathbb{R}(N)$ the range of generator α and write $\dot{x}$ instead of $\alpha(x)$ for $x \in \mathbb{R}$. $\mathbb{R}(N)$ is called Non-Newtonian real line. Non-Newtonian arithmetic operations on $\mathbb{R}(N)$ are represented as follows:

$\alpha - addition$ $\quad \dot{x} \dot{+} \dot{y} = \alpha(x+y)$
$\alpha - subtraction$ $\quad \dot{x} \dot{-} \dot{y} = \alpha(x-y)$
$\alpha - multiplication$ $\quad \dot{x} \dot{\times} \dot{y} = \alpha(x \times y)$
$\alpha - division$ $\quad \dot{x} \dot{/} \dot{y} = \alpha(x \div y)$
$\alpha - order$ $\quad \dot{x} \dot{<} \dot{y} \left(\dot{x} \dot{\leq} \dot{y}\right) \Leftrightarrow x < y \, (x \leq y)$.

Closed intervals are represented by

$$\begin{aligned} \left[\dot{a}, \dot{b}\right]_N &= \left\{\dot{x} \in \mathbb{R}(N) : \dot{a} \dot{\leq} \dot{x} \dot{\leq} \dot{b}\right\} \\ &= \left\{\dot{x} \in \mathbb{R}(N) : a \leq x \dot{\leq} \; b\right\} = \alpha([a,b]). \end{aligned}$$

Likewise open and semi-open intervals can be represented. Additionally the midpoint of $\left[\dot{a}, \dot{b}\right]_N$ is $\dot{c} = \left(\dot{a} \dot{+} \dot{b}\right) \dot{/} \dot{2}$ as in $\mathbb{R}$. Indeed since

$$\begin{aligned} \dot{c} \dot{-} \dot{a} &= \alpha(c-a) = \alpha\left\{\alpha^{-1}\left[\left(\dot{a} \dot{+} \dot{b}\right) \dot{/} \dot{2}\right] - a\right\} \\ &= \alpha\left\{\alpha^{-1}\left[\alpha((a+b) \div 2)\right] - a\right\} \\ &= \alpha\left\{\left[(a+b) \div 2\right] - a\right\} \\ &= \alpha\left\{(b-a) \div 2\right\} \end{aligned}$$

and

$$\begin{aligned} \dot{b} \dot{-} \dot{c} &= \alpha(b-c) = \alpha\left\{b - \alpha^{-1}\left[\left(\dot{a} \dot{+} \dot{b}\right) \dot{/} \dot{2}\right]\right\} \\ &= \alpha\left\{b - \alpha^{-1}\left[\alpha((a+b) \div 2)\right]\right\} \\ &= \alpha\left\{b - \left[(a+b) \div 2\right]\right\} \\ &= \alpha\left\{(b-a) \div 2\right\}, \end{aligned}$$

the equality $c \dot{-} \dot{a} = \dot{b} \dot{-} \dot{c}$ holds.

The α-absolute value of a number $\dot{x}$ in $\mathbb{R}(N)$ is defined as $\alpha(|x|)$ and is denoted by $\left|\dot{x}\right|_N$. Accordingly, we write

$$\left|\dot{x}\right|_N = \begin{cases} \dot{x}, & if \ \dot{x} \dot{>} \dot{0} \\ \dot{0}, & if \ \dot{x} = \dot{0} \\ \dot{0} \dot{-} \dot{x}, & if \ \dot{x} \dot{<} \dot{0} \end{cases} = \alpha(|x|).$$

If a function $d_N : \mathbb{R}(N) \times \mathbb{R}(N) \to \mathbb{R}^+(N)$ is defined by $d_N\left(\dot{x}, \dot{y}\right) = \left|\dot{x} \dot{-} \dot{y}\right|_N = \alpha(|x-y|)$, then it is known that the pair $(\mathbb{R}(N), d_N)$ is non-Newtonian metric space. Here non-Newtonian metric axioms are as follows:

(NM1) $d_N\left(\dot{x}, \dot{y}\right) = \dot{0}$ if and only if $\dot{x} = \dot{y}$,
(NM2) $d_N\left(\dot{x}, \dot{y}\right) = d_N\left(\dot{y}, \dot{x}\right)$,

(NM3) $d_N\left(\dot{x},\dot{y}\right) \leq d_N\left(\dot{x},\dot{z}\right) + d_N\left(\dot{z},\dot{y}\right)$.

The $\dot{\varepsilon}$-neighborhood of a point $\dot{x}$ in $\mathbb{R}(N)$ is denoted by $N\left(\dot{x},\dot{\varepsilon}\right)$ and is defined by

$$\begin{aligned} N\left(\dot{x},\dot{\varepsilon}\right) &= \left\{\dot{y} \in \mathbb{R}(N) : \left|\dot{x} \dot{-} \dot{y}\right|_N \dot{<} \dot{\varepsilon}\right\} \\ &= \left\{\dot{y} \in \mathbb{R}(N) : \alpha^{-1}\left(\left|\dot{x} \dot{-} \dot{y}\right|_N\right) < \alpha^{-1}\left(\dot{\varepsilon}\right)\right\} \\ &= \left\{\dot{y} \in \mathbb{R}(N) : \alpha^{-1}\left[\alpha\left(|x-y|\right)\right] < \varepsilon\right\} \\ &= \left\{\dot{y} \in \mathbb{R}(N) : |x-y| < \varepsilon\right\}. \end{aligned}$$

Let $\dot{x}$ be a point in $\mathbb{R}(N)$ and A be a subset in $\mathbb{R}(N)$. If there is a point belonging to A, different from $\dot{x}$ in all interval $\left(\dot{a},\dot{b}\right)_N$ containing $\dot{x}$, then $\dot{x}$ is called a limit point of A. All neigborhood of a limit point of a set contains an infinite number of points of this set, as in $\mathbb{R}$. If there is two numbers as $\dot{a}$ and $\dot{b}$ in $\mathbb{R}(N)$ with $\dot{a} \leq \dot{x} \leq \dot{b}$ for all $\dot{x} \in A$, then it is said to be a bounded subset in $\mathbb{R}(N)$.

Grossman and Katz introduced the non-Newtonian Calculus consisting of the branches of geometric, anageometric and bigeometric calculus etc in [7]. Following them, Çakmak and Başar constructed the field $\mathbb{R}(N)$ of non-Newtonian real numbers and the concept of non-Newtonian metric in [6]. They also gave the triangle and Minkowski's inequalities in the sense of non-Newtonian Calculus, additiomally studied the known and various sequence spaces.

A multiplicative calculus is a system with two multiplicative operators, called a "multiplicative derivative" and a "multiplicative integral", which are inversely related in a manner analogous to the inverse relationship between the derivative and integral in the classical calculus of Newton and Leibniz. The multiplicative calculi provide alternatives to the classical calculus, which has an additive derivative and an additive integral. There are infinitely many non-Newtonian multiplicative calculi, including the geometric calculus and the bigeometric calculus. These calculi all have a derivative and integral that is not a linear operator.

Bashirov et al. have studied on multiplicative calculus and gave the results with applications corresponding to the well-known properties of derivatives and integrals in classical calculus in [2]. Again, Uzer has extended the multiplicative calculus to the complex-valued functions and demonstrated some analogies between the multiplicative complex calculus and classical calculus in [8]. We take [5] as reference to the basic concepts and theorems of real analysis.

2 Main Results

Theorem 2.1. *(Bolzano-Weierstrass Theorem). All bounded infinite subset E of $\mathbb{R}(N)$ has at least one limit point.*

Proof. Let $E \subset \mathbb{R}(N)$ be an arbitrary bounded infinite subset. Then there is two numbers as $\dot{a}$ and $\dot{b}$ in $\mathbb{R}(N)$ with $E \subset \left[\dot{a},\dot{b}\right]_N$. We say $\dot{c}$ to midoint of $\left[\dot{a},\dot{b}\right]_N$. In this case we know that $\dot{c} = \left(\dot{a} \dot{+} \dot{b}\right) \dot{/} \dot{2} = \alpha\left[(a+b) \div 2\right]$. Since E is infinite, at least one represented by $\left[\dot{a}_1,\dot{b}_1\right]_N$ of the intervals $\left[\dot{a},\dot{c}\right]_N$ and $\left[\dot{c},\dot{b}\right]_N$ contains the infinite members of E. If $\dot{c}_1$is midpoint of $\left[\dot{a}_1,\dot{b}_1\right]_N$, then the lenghts of $\left[\dot{a}_1, \dot{c}_1\right]_N$ and $\left[\dot{c}_1, \dot{b}_1\right]_N$ is the same, and we also have

$$\begin{aligned}
\dot{c}_1 \dot{-} \dot{a}_1 &= \dot{b}_1 \dot{-} \dot{c}_1 = \alpha\left\{(b-c) \div 2\right\} \\
&= \alpha\left(\frac{b-\alpha^{-1}\left(\alpha\left[\frac{a+b}{2}\right]\right)}{2}\right) \\
&= \alpha\left(\frac{b-\frac{a+b}{2}}{2}\right) = \alpha\left(\frac{b-a}{2^2}\right)
\end{aligned}$$

or

$$\begin{aligned}
\dot{c}_1 \dot{-} \dot{a}_1 &= \dot{b}_1 \dot{-} \dot{c}_1 = \alpha\left\{(c-a) \div 2\right\} \\
&= \alpha\left(\frac{\alpha^{-1}\left(\alpha\left[\frac{a+b}{2}\right]\right)-a}{2}\right) \\
&= \alpha\left(\frac{\frac{a+b}{2}-a}{2}\right) = \alpha\left(\frac{b-a}{2^2}\right).
\end{aligned}$$

Since E is infinite, at least one represented by $\left[\dot{a}_2, \dot{b}_2\right]_N$ of the intervals $\left[\dot{a}_1, \dot{c}_1\right]_N$ and $\left[\dot{c}_1, \dot{b}_1\right]_N$ contains the infinite members of E and has the leght

$$\alpha\left(\frac{b-a}{2^2}\right).$$

Infinitely repeating this process, we obtain a system of nested intervals in $\mathbb{R}(N)$, having the following conditions:

(i) The intervals $\left[\dot{a}_n, \dot{b}_n\right]_N$ for all $n \in \mathbb{N}$ contain the infinite members of E,
(ii) $\left[\dot{a}, \dot{b}\right]_N \supset \left[\dot{a}_1, \dot{b}_1\right]_N \supset \left[\dot{a}_2, \dot{b}_2\right]_N \supset \cdots \supset \left[\dot{a}_n, \dot{b}_n\right]_N \supset \cdots$,
(iii) The lenght of the intervals $\left[\dot{a}_n, \dot{b}_n\right]_N$ for all $n \in \mathbb{N}$ is

$$\alpha\left(\frac{b-a}{2^n}\right).$$

Since α is continuous, $\alpha\left(\frac{b-a}{2^n}\right) \to 0$ as $n \to \infty$. Hence there exists a point $\dot{x}_0 = \alpha(x_0)$, contained in all of the intervals $\left[\dot{a}_n, \dot{b}_n\right]_N$, such that

$$\lim_{n\to\infty} \dot{a}_n = \lim_{n\to\infty} \dot{a}_n = \dot{x}_0.$$

This point x_0 is obtained from a well-known theorem in the theory of limits. Indeed the following properties hold

(i) The intervals $[a_n, b_n]$ for all $n \in \mathbb{N}$ contain the infinite members of bounded and infinite set $\alpha^{-1}(E)$ in $\mathbb{R}$.
(ii) $[a, b] \supset [a_1, b_1] \supset [a_2, b_2] \supset \cdots \supset [a_n, b_n] \supset \cdots$,
(iii) The lenght of the interval $[a_n, b_n]$ for all $n \in \mathbb{N}$ is

$$\frac{b-a}{2^n}.$$

So there is a point x_0 in $\alpha^{-1}(E)$, contained in all of the intervals $[a_n, b_n]$, such that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = x_0.$$

Finally $\dot{x}_0$is a limit point of E. Indeed if $\left(\dot{\alpha},\dot{\beta}\right)_N$ is an arbitrarily open interval in $\mathbb{R}(N)$ concerning $\dot{x}_0$, then there obviously is sufficiently large $n\in\mathbb{N}$ with $\left[\dot{a}_n,\dot{b}_n\right]_N\subset\left(\dot{\alpha},\dot{\beta}\right)_N$. This completes the proof.

The second version of Bolzano-Weierstrass Theorem, mentioned above, is as follows. □

Theorem 2.2. *From every bounded sequence $\left(\dot{x}_n\right)$ in $\mathbb{R}(N)$, it can be obtained a bounded subsequence $\left(\dot{x}_{n_k}\right)$.*

Proof. Let E be the set consisting of terms of $\left(\dot{x}_n\right)$. If E is bounded, then what is requested is obtained immediately. If E is infinite, then Theorem 1 can be applied to it. Let $\dot{x}_0$ be a limit point of E. Then x_0 is a limit point of $\alpha^{-1}(E)$ and we can select from $\alpha^{-1}(E)$ a subsequence $(x_{n_k})=\alpha^{-1}\left(\dot{x}_{n_k}\right)$ of (x_n) convercing to x_0. In this case, evidently $\lim_{n\to\infty}\dot{x}_{n_k}=\dot{x}_0$. □

We now introduce the provisions in $\mathbb{R}(N)$ of some well-known definitions and notations in real analysis:

1. The set of all limit points of $E\subset\mathbb{R}(N)$ is denoted by $E^{'}$.
2. If $E^{'}\subset E$, the set E is said to be closed.
3. If $E\subset E^{'}$, the set E is said to be dense in itself.
4. If $E=E^{'}$, the set E is said to be perfect.
5. The set $E\cup E^{'}$ is called the closure of the set E and is denoted by $\bar{E}$.

Example 2.3. *(i) For the subset $E=\left\{\dot{1},\frac{\dot{1}}{2},\frac{\dot{1}}{3},\cdots,\frac{\dot{1}}{n},\cdots\right\}\subset\mathbb{R}(N)$, we have $E^{'}=\{0\}$, because $\frac{1}{n}\to 0$ as $n\to\infty$. So E is neither closed nor dense in itself. (ii) If $E=\left(\dot{a},\dot{b}\right)_N$, then $\bar{E}=\left[\dot{a},\dot{b}\right]_N$. Indeed we have $\alpha^{-1}(E)=(a,b)$ and so $\overline{\alpha^{-1}(E)}=[a,b]$. In that case E is dense in itself. (iii) Since $\bar{E}=\left[\dot{a},\dot{b}\right]$ if $E=\left[\dot{a},\dot{b}\right]$, E is perfect. (iv) If $E=\mathbb{R}(N)$, then E is perfect. (v) Let $E=\left\{\dot{x}_1,\dot{x}_2,\cdots,\dot{x}_n\right\}$ be a finite subset in $\mathbb{R}(N)$. Then the neigborhood $\left(\dot{x}_i\dot{-}\dot{c},\dot{x}_i\dot{+}\dot{c}\right)_N=\left\{\dot{y}\in\mathbb{R}(N):\left|\dot{x}-\dot{y}\right|\dot{<}\ \dot{c}\right\}$ of any $\dot{x}_i\in\mathbb{R}(N)$, $i=1,2,\cdots,n$ does not contain any element of E. Here c is as follows:*

$$\dot{c}=\begin{cases}\left|\dot{x}_1\dot{-}\dot{x}_2\right|_N, & if\quad i=1\\ \min\left(\left|\dot{x}_i\dot{-}\dot{x}_{i-1}\right|_N,\left|\dot{x}_i\dot{-}\dot{x}_{i-1}\right|_N\right), & if\quad i=2,\cdots,n\\ \left|\dot{x}_{n-1}\dot{-}\dot{x}_n\right|_N, & if\quad i=1\end{cases}.$$

Accordingly $E^{'}=\phi$, namely empty set.

Theorem 2.4. *For an arbitrary non-empty subset $E\subset\mathbb{R}(N)$, $E^{'}$ is closed.*

Proof. Let $\dot{a}$ be a limit point of $E^{'}$. We select an interval $\left(\dot{\alpha},\dot{\beta}\right)_N$ containing the point $\dot{a}$. There exists one $\dot{x}\in E^{'}$ in $\left(\dot{\alpha},\dot{\beta}\right)_N$ with $\dot{x}\neq\dot{a}$. This implies that $\left(\dot{\alpha},\dot{\beta}\right)_N$ contains an infinite number of point E. Hence $\dot{a}\in E^{'}$. □

Theorem 2.5. *(i) If $E\subset F$ in $\mathbb{R}(N)$, then $E^{'}\subset F^{'}$. (ii) $(E\cup F)^{'}=E^{'}\cup F^{'}$for any subsets $E,F\subset\mathbb{R}(N)$.*

Proof. (i) If $\dot{a} \in E^{'}$, then an arbitrary open interval $\left(\dot{\alpha}, \dot{\beta}\right)_N$ containing $\dot{a}$ include an infinite number of points of E and, by the hypothesis, of F. (ii) The inclusion $E^{'} \cup F^{'} \subset (E \cup F)^{'}$ is apparent from (i). If $\dot{a} \in (E \cup F)^{'}$, then an arbitrary open interval $\left(\dot{\alpha}, \dot{\beta}\right)_N$ containing $\dot{a}$ include an infinite number of points of E or F. Thus $\dot{a} \in E^{'} \cup F^{'}$ and accordingly $(E \cup F)^{'} \subset E^{'} \cup F^{'}$.

Corollary 2.6. *(i) The closure of a set F in $\mathbb{R}(N)$ is closed. (ii) A set F in $\mathbb{R}(N)$ is closed if and only if $E = \bar{\bar{E}}$. (iii) The composition of two closed sets in $\mathbb{R}(N)$ is closed.*

□

Remark 2.1. The composition of an infinite number of closed sets in $\mathbb{R}(N)$ is closed.

For example, let $F_n = \left[\frac{\dot{1}}{n}, \dot{1}\right]_N$, $n = 1, 2, \cdots$. Since

$$\left[\frac{\dot{1}}{n}, \dot{1}\right]_N = \left\{\dot{x} : \ \alpha^{-1}\left(\frac{\dot{1}}{n}\right) \leq x \leq \alpha^{-1}\left(\dot{1}\right)\right\} = \left\{\dot{x} : \frac{1}{n} \leq x \leq 1\right\} = \alpha\left(\left[\frac{1}{n}, 1\right]\right),$$

we have

$$\cup_{n=1}^{\infty}\left[\frac{\dot{1}}{n}, \dot{1}\right]_N = \cup_{n=1}^{\infty}\alpha\left(\left[\frac{1}{n}, 1\right]\right) = \alpha\left(\cup_{n=1}^{\infty}\left[\frac{1}{n}, 1\right]\right) = \alpha\left((0, 1]\right) = \left(\dot{0}, \dot{1}\right]_N.$$

The proof of the following lemma and two theorems is as in $\mathbb{R}$.

Theorem 2.7. *The intersection of an arbitrary family of closed sets in $\mathbb{R}(N)$ is closed.*

Lemma 2.8. *Let E be a bounded above(below) subset in $\mathbb{R}(N)$ and $\beta = \sup E$ $(\alpha = \inf E)$. Then $\beta \in \bar{E}$ $(\alpha \in \bar{E})$.*

Theorem 2.9. *A closed subset which is bounded above(below) in $\mathbb{R}(N)$ has a right (left) end point.*

Definition 2.1. Let E be a subset in $\mathbb{R}(N)$ and P be a family of open intervals. If for every $x \in E$, there exists an interval $\delta \in P$ such that $x \in \delta$, then we say the set E is covered by P.

Theorem 2.10. *(Heine-Borel Theorem). If the closed bounded subset F of $\mathbb{R}(N)$ is covered by an infinite family P of open intervals in $\mathbb{R}(N)$, then it is possible to select a finite subfamily P^* of P which also covers the set F.*

Proof. Assume that it is not possible to select a finite subfamily P^* of P which also covers the set F. We choose a closed interval $\left[\dot{a}, \dot{b}\right]_N$ in $\mathbb{R}(N)$ with $F \subset \left[\dot{a}, \dot{b}\right]_N$ and we construct a sequence of nested closed intervals as follows:

(i) Each of the sets $\left[\dot{a}_n, \dot{b}_n\right]_N \cap F$ with $n \in \mathbb{N}$ has the infinite members,

(ii) None of the sets $\left[\dot{a}_n, \dot{b}_n\right]_N \cap F$ with $n \in \mathbb{N}$ is covered with a finite number of P,

(iii) $\left[\dot{a}, \dot{b}\right]_N \supset \left[\dot{a}_1, \dot{b}_1\right]_N \supset \left[\dot{a}_2, \dot{b}_2\right]_N \supset \cdots \supset \left[\dot{a}_n, \dot{b}_n\right]_N \supset \cdots$,

(iv) The lenght of the intervals $\left[\dot{a}_n, \dot{b}_n\right]_N$ for all $n \in \mathbb{N}$ is

$$\alpha\left(\frac{b-a}{2^n}\right).$$

There exists exactly one $\dot{x}_0$belonging to all of the closed sets $\left[\dot{a}_n, \dot{b}_n\right]_N$ with $n \in \mathbb{N}$. We clearly

$$\lim_{n\to\infty} \dot{a}_n = \lim_{n\to\infty} \dot{b}_n = \dot{x}_0.$$

Again we can select a point $\dot{x}_n$such that

$$\dot{a}_n \le \dot{x}_n \le \dot{b}_n$$

from the set $\left[\dot{a}_n, \dot{b}_n\right]_N \cap F$ for all $n \in \mathbb{N}$ with $\dot{x}_n \ne \dot{x}_m$where $n \ne m$. Then obviously $\lim_{n\to\infty} \dot{x}_n = \dot{x}_0$and $\dot{x}_0 \in F$, since F is closed.

Since F is covered by the family P, there is an open interval δ_0 in P such that $\dot{x}_0 \in \delta_0$. Clearly $\left[\dot{a}_n, \dot{b}_n\right]_N \cap F \subset \delta_0$ for sufficiently large n. Thus the set $\left[\dot{a}_n, \dot{b}_n\right]_N \cap F$ is covered by a single interval in P. This is a contradiction. □

Remark 2.2. If we neglect one of the conditions F, then this theorem can not be valid.

We consider the set $\dot{\mathbb{N}} = \alpha(\mathbb{N})$ of all non-Newtonian natural numbers. It can be easily seen that $\dot{\mathbb{N}}$ is closed, but unbounded.

3 CONCLUSIONS

In this paper, the authors introduced and examined for non-Newtonian analysis some fundamenal concepts and usual algeabric and topologic properties related to this concepts. Hereafter, using these done one can introduce and examine for non-Newtonian analysis the concepts and properties related to the sequences and functions.

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