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Uniqueness of q-Difference Value on Sharing One

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Abstract

In this paper, we investigate uniqueness problems of q-difference transcendental meromorphic functions with zero order sharing one value. We obtain some results on q-difference, which extend many previous results.

Keywords: Uniqueness; meromorphic functions; q-difference; share value; zero order.

2010 Mathematics Subject Classification: 30D35.

1 Introduction and Main Results

When investigating uniqueness problems of q-difference functions, we always use kind of function, which is meromorphic in the whole complex plane except at possible poles. In this paper, we define this as a meromorphic function. If no poles occur, it reduces to an entire function. Let q be non-zero complex constant in what follows, and q-difference of f (z) be defined by f (qz). We assume the reader is familiar with the standard notations and results such as the proximity function m(r, f), counting function N (r, f), characteristic function T (r, f), the elementary Nevanlinna theory, see, e.g., [1]. We denote by S(r, f) any quantity satisfying $S(r, f) = o(T(r, f))$, as $r \to \infty$ possibly outside a set of log logarithmic density 0.

We define that f, g are meromorphic and share a value a IM (ignoring multiplicities) if $f - a$ and $g - a$ have

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the same zeros. While f − a and g – a have not only the same zeros but also the same multiplicities, we say that f and g share a CM (counting multiplicities).

In 1959, Hayman [2] proved that $f^n f'$ takes every non-zero complex value infinitely often if n ≥ 3, when f is a transcendental meromorphic function, and n is a positive integer. In particular, he proved the following famous theorem.

Theorem A. If f is a transcendental meromorphic function, n(≥ 5) is a positive integer, and a(≠ 0) is a constant, then $f'(z) - af(z)^n$ takes every finite complex value b infinitely often. If f is a transcendental entire function, then n ≥ 3 or n ≥ 2, when b = 0 also holds.

Liu [3] considered the difference counterpart of Theorem A, and proved the following result.

Theorem B. If f is a transcendental meromorphic function with finite order, period is not c, and a is a non-zero constant, n ≥ 8, then $f(z)^n + a(f(z+c)-f(z)) - s(z)$ has infinitely many zeros.

In this paper, on the basis of theorems A and B, we study uniqueness problems of two q-difference polynomials sharing one value and obtain the following results.

Theorem 1.1. Let f and g be two transcendental meromorphic functions with zero order, $a \in C \setminus \{0\}$, $b \in C$, and let n be a positive integer satisfying n ≥ 26. Assume that the functions

$$\phi_f := \frac{-af(qz)+b}{f^n} \quad \text{and} \quad \phi_g := \frac{-ag(qz)+b}{g^n} \tag{1.1}$$

Share the value 1 IM. Then

$$\phi_f = \phi_g \tag{1.2}$$

or

$$\phi_f . \phi_g = 1 \tag{1.3}$$

Theorem 1.2. Let f and g be two transcendental entire functions with zero order, $a \in C \setminus \{0\}$, $b \in C$, and let n be a positive integer satisfying n ≥ 14. Assume that the functions ϕ_f and ϕ_g defined as in (1.1) share the value 1 IM. Then (1.2) holds.

When considering the particularity of meromorphic functions, we have two relatively simple results as follows.

Theorem 1.3. Let f and g be two transcendental meromorphic functions with zero order, $a \in C \setminus \{0\}$, $b \in C$, and let n be a positive integer satisfying n ≥ 14. Assume that the functions ϕ_f and ϕ_g defined as in (1.1) share the value 1CM. Then (1.2) or (1.3) holds.

Theorem 1.4. Let f and g be two transcendental entire functions with zero order, $a \in C \setminus \{0\}$, $b \in C$, and let n be a positive integer satisfying n ≥ 8. Assume that the functions ϕ_f and ϕ_g defined as in (1.1) share the value 1 CM. Then (1.2) holds.

2 Lemmas

In this section, we present some lemmas which play an important role in the following proofs. The following q-shift difference analogue of the logarithmic derivative lemma is very important when we consider ring q-shift difference polynomials.

Lemma 2.1 ([4], Theorem 2.1]). Let f(z) be a meromorphic function of zero order.
Then

$$m(r, \frac{f(qz+c)}{f(z)}) = o(T(r,f))$$

On a set of logarithmic density 1.

The next two lemmas are essential in our proofs, which offer us the way to estimate the characteristic function and counting function of f(qz), see the theorems 1.1 and 1.3 in [5].

Lemma 2.2 Let f(z) be a nonconstant zero order meromorphic function, and $q \in C \setminus \{0\}$,. Then

$$T(r, f(qz)) = T(r, f(z)) + S(r, f)$$

on a set of logarithmic density 1.

Lemma 2.3 Let f(z) be a nonconstant zero order meromorphic function, and $q \in C \setminus \{0\}$,. Then

$$N(r, f(qz)) = (1 + o(1))N(r, f(z))$$

on a set of lower logarithmic density 1.

Now, we introduce some notations. Let F and G be two nonconstant meromorphic functions, $a \in C \cup \infty$. Suppose that z_0 is an a-point both of F of order p and of G of order q. We denote by $N_A(r,a)$ the counting function of the common a-point of F and G satisfying the condition A, where each point is counted once. For example, $N_{p=1=q}(r,a)$ denotes the counting function of those common simple a-point of F and G, as well as $N_{p>q}(r,a)$ of F and G with the orders p>q. Furthermore, separately defining $N_{p)}(r, \frac{1}{F-a})$ and $N_{(p+1}(r, \frac{1}{F-a})$ is the counting function of the zeros of F-a with the orders less than or equal to p and with the orders larger than p, in which each point is counted once. However, $N_p(r, \frac{1}{f-a})$ denotes the counting function of the zeros of f-a where m-fold zeros are counted m times if m$\leq$ p and p times if m>p. Now we denote

$$N^*(r,a) = N_{(2}(r, \frac{1}{F-a}) + N_{(2}(r, \frac{1}{G-a}) - N_{p>1,q>1}(r,a) \geq 0 \qquad (2.1)$$

The auxiliary function H in the following lemma plays an important role on solving two meromorphic functions, which share one finite value.

Lemma 2.4 ([6], Theorem 3]). Let

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right) \tag{2.2}$$

where F and G are two nonconstant meromorphic functions. If H $\not\equiv$0**,** then

$$N_{p=1=q}(r,1) \le N(r,H) + S(r,F) + S(r,G)$$

When considering two nonconstant meromorphic functions F, G that share at least one finite value IM, we could find the following lemma play a key role. The proof of the lemma in [7] is submitted, but for the convenience of readers, we list here again. In the original paper, S(r,F) denotes any quantity of S(r,F)=o(T(r,F)), as $r \to \infty$ possibly outside a set of finite linear measure. So it holds when S(r,F)=o(T(r,F)), as $r \to \infty$ possibly outside a set of logarithmic density 0.

Lemma 2.5 ([7]). Let F and G be two nonconstant meromorphic functions. If F and G share 1 IM, then one of the following three cases holds:

(1)

$$\begin{aligned} &T(r,F) + T(r,G) \\ &\le 3\{\overline{N}(r,1/F) + \overline{N}(r,1/G) + \overline{N}(r,F) + \overline{N}(r,G)\} \\ &+ 2\{N_2(r,1/F) + N_2(r,1/G) + N_2(r,F) + N_2(r,G) \\ &- N_{p>1,q>1}(r,0) - N_{p>1,q>1}(r,\infty)\} + S(r,F) + S(r,G)\,; \end{aligned} \tag{2.3}$$

(2) $FG = 1$

(3) $F = G$

where $N_2(r,1/F)$ denotes the counting function of zeros of F such that simple zeros are counted once and multiple zeros twice.

Proof. Let H be given as in (2.2). Then

$$\begin{aligned} N(r,H) &\le N^*(r,0) + N^*(r,\infty) + N_{p>q}(r,1) + N_{p<q}(r,1) \\ &+ N_0(r,1/F') + N_0(r,1/G') \end{aligned} \tag{2.4}$$

where $N_0(r,1/F')$ denotes the counting function corresponding to the zeros of F' which are not the zeros of F and F-1, and correspondingly for G'. From Lemma 2.4 and (2.4), if $H \ne 0$, we have

$$\begin{aligned} &\overline{N}\left(r,\frac{1}{F-1}\right) + \overline{N}\left(r,\frac{1}{G-1}\right) \\ &= 2N_{p=q=1}(r,1) + 2N_{p>q}(r,1) + 2N_{p<q}(r,1) + 2N_{p=q>1}(r,1) \end{aligned}$$

$$\begin{aligned}
&\leq N_{p=q=1}(r,1) + N_{p>q}(r,1) + N(r,1/(G-1)) \\
&\leq N_{p=q=1}(r,1) + N_{p>q}(r,1) + T(r,G) + O(1) \\
&\leq N^*(r,0) + N^*(r,\infty) + 2N_{p>q}(r,1) + N_{p<q}(r,1) + T(r,G) \\
&+ N_0(r,1/F') + N_0(r,1/G') + S(r,F) + S(r,G)
\end{aligned}$$

From the second fundamental theorem, we have

$$T(r,F) \leq \overline{N}(r,1/F) + \overline{N}(r,F) + \overline{N}(r,\frac{1}{F-1}) - N_0(r,F') + S(r,F),$$

$$T(r,G) \leq \overline{N}(r,1/G) + \overline{N}(r,G) + \overline{N}(r,\frac{1}{G-1}) - N_0(r,G') + S(r,G).$$

Firstly, noting that

$$\begin{aligned}
&N_{p>q}(r,1) \leq N(r,F/F') \leq N(r,F'/F) + S(r,F) \\
&\leq \overline{N}(r,1/F) + \overline{N}(r,F) + S(r,F)
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
&N_{p<q}(r,1) \leq N(r,G/G') \leq N(r,G'/G) + S(r,G) \\
&\leq \overline{N}(r,1/G) + \overline{N}(r,G) + S(r,G)
\end{aligned} \tag{2.6}$$

Combining the above three inequalities with (2.5) and (2.6), we obtain

$$\begin{aligned}
T(r,F) + T(r,G) &\leq N_2(r,1/F) + N_2(r,1/G) - N_{p>1,q>1}(r,0) + N_2(r,F) \\
&+ N_2(r,G) - N_{p>1,q>1}(r,\infty) + 2N_{p>q}(r,1) + N_{p<q}(r,1) \\
&+ T(r,G) + S(r,F) + S(r,G),
\end{aligned}$$

Which

$$\begin{aligned}
T(r,F) &\leq N_2(r,1/F) + N_2(r,1/G) - N_{p>1,q>1}(r,0) + N_2(r,F) \\
&+ N_2(r,G) - N_{p>1,q>1}(r,\infty) + 2N_{p>q}(r,1) + N_{p<q}(r,1) \\
&+ S(r,F) + S(r,G),
\end{aligned}$$

Similarly,

$$\begin{aligned}
T(r,G) &\leq N_2(r,1/F) + N_2(r,1/G) - N_{p>1,q>1}(r,0) + N_2(r,F) \\
&+ N_2(r,G) - N_{p>1,q>1}(r,\infty) + N_{p>q}(r,1) + 2N_{p<q}(r,1) \\
&+ S(r,F) + S(r,G),
\end{aligned}$$

So we have

$$T(r,F)+T(r,G)\le 2\{N_2(r,1/F)+N_2(r,1/G)-N_{p>1,q>1}(r,0)+N_2(r,F)+N_2(r,G)-N_{p>1,q>1}(r,\infty)\}+3\{N_{p>q}(r,1)+N_{p<q}(r,1)\}+S(r,F)+S(r,G); \tag{2.7}$$

The conclusion (1) follows by inserting the last two inequalities into (2.7).

If H$\neq 0$, by integration, we get from (2.2) that

$$\frac{1}{F-1}=\frac{A}{G-1}+B, \tag{2.8}$$

where A($\neq 0$) and B are constants. From (2.8) we have

$$F=\frac{(1+B)G+A-B-1}{BG+A-B},\quad G=\frac{(B-A)F+A-B-1}{BF-(B+1)} \tag{2.9}$$

We discuss the following three cases.

Case 1. Suppose that B$\neq$0,-1. If B$\neq$ A, we get from (2.9) that $\overline{N}\left(r,1/\left(F-\frac{B+1}{B}\right)\right)=\overline{N}(r,G)$, $\overline{N}\left(r,1/\left(G+\frac{A-B}{B}\right)\right)=\overline{N}(r,F)$. From the second fundamental theorem, we have

$$T(r,F)\le \overline{N}(r,1/F)+\overline{N}(r,F)+\overline{N}\left(r,1/\left(F-\frac{B+1}{B}\right)\right)+S(r,F)$$
$$\le \overline{N}(r,1/F)+\overline{N}(r,F)+\overline{N}(r,G)+S(r,F)$$

and

$$T(r,G)\le \overline{N}(r,1/G)+\overline{N}(r,G)+\overline{N}\left(r,1/\left(G+\frac{A-B}{B}\right)\right)+S(r,G)$$
$$\le \overline{N}(r,1/G)+\overline{N}(r,G)+\overline{N}(r,F)+S(r,G)$$

Thus

$$T(r,F)+T(r,G)\le \overline{N}(r,1/F)+\overline{N}(r,1/G)+2\overline{N}(r,F)+2\overline{N}(r,G)+S(r,G)+S(r,F)$$

The conclusion (1) holds. If B=A, rewrite (2.9) as

$$F=\frac{(1+B)G-1}{BG},\quad G=\frac{-1}{BF-(B+1)}$$

Noting that $\overline{N}(r,1/\left(F-\frac{B+1}{B}\right)) = \overline{N}(r,G)$ and $\overline{N}(r,1/\left(G-\frac{1}{B+1}\right)) = \overline{N}(r,F)$

The conclusion (1) holds by the second fundamental theorem and the same arguments as above.

Case 2. Suppose that B=0. From (2.9) we have

$$F = \frac{(G+A-1)}{A}, \quad G = AF-(A-1) \tag{2.10}$$

If A $\neq$ 1, by (2.10), we obtain $\overline{N}(r, \frac{1}{G+A-1}) = \overline{N}(r,1/F)$ and $\overline{N}(r, \frac{1}{F-\frac{A-1}{A}})$

$= \overline{N}(r,1/G)$. The conclusion (1) follows again by the second fundamental theorem. If A=1, then F=G, which is the conclusion (3).

Case 3. Suppose that B=-1. (2.9) yields

$$F = \frac{(-A)}{G-A+B}, \quad G = \frac{(1+A)F-A}{F} \tag{2.11}$$

If A $\neq$ -1, we obtain the conclusion (1) by the same reasoning discussed in Case 2. If A=-1, we get FG=1 from (2.11).

The following corresponding result is about two meromorphic functions sharing 1 CM.

Lemma 2.6 ([8], Lemma [3]). Let F and G be two nonconstant meromorphic functions. If F and G share 1 CM, then one of the following three cases holds:

(1) $\max\{T(r,F), T(r,G)\} \leq N_2(r,1/F) + N_2(r,1/G) + N_2(r,F) + N_2(r,G) + S(r,F) + S(r,G)$;

(2) $FG = 1$;

(3) $F = G$,

where $N_2(r,1/F)$ denotes the counting function of zeros of F such that simple zeros are counted once and multiple zeros twice.

3 Proofs of the Theorems

In this section, the proofs of our results are given.

Proof of Theorem 1.1. The first main theorem and Lemma 2.2 gives

$$nT(r,f) = T(r,1/f^n) + O(1)$$

$$\leq T\left(r, \frac{-af(qz)+b}{f^n}\right) + T\left(r, \frac{1}{-af(qz)+b}\right) + O(1)$$
$$\leq T(r, \phi_f) + T(r, f) + S(r, f), \tag{3.1}$$

and so

$$(n-1)T(r, f) \leq T(r, \phi_f) + S(r, f) \tag{3.2}$$

Similarly,

$$(n-1)T(r, g) \leq T(r, \phi_g) + S(r, g) \tag{3.3}$$

Then ϕ_f is nonconstant, so is ϕ_g. From the conditions of Theorem 1.1, we know that ϕ_f and ϕ_g share 1 IM. By (1.1) and Lemma 2.3, we get

$$N_2(r, 1/\phi_f) \leq N_2\left(r, \frac{1}{-af(qz)+b}\right) + 2\overline{N}(r, f),$$

and

$$N_2(r, \phi_f) \leq N(r, f(qz)) + 2\overline{N}(r, 1/f) + S(r, f)$$

$$\leq N(r, f) + 2\overline{N}(r, 1/f) + S(r, f)$$

Similarly, we get

$$N_2(r, 1/\phi_g) \leq N_2\left(r, \frac{1}{-ag(qz)+b}\right) + 2\overline{N}(r, g),$$

$$N_2(r, \phi_g) \leq N(r, g) + 2\overline{N}(r, 1/g) + S(r, g).$$

Suppose that (2.3) in Lemma 2.5 holds. The above two inequalities with Lemmas 2.2 and 2.3 yield

$$T(r, \phi_f) + T(r, \phi_g) \leq 3\{\overline{N}(r, 1/\phi_f) + \overline{N}(r, \phi_f) + \overline{N}(r, 1/\phi_g) + \overline{N}(r, \phi_g)\}$$

$$+ 2\{N_2(r, 1/\phi_f) + N_2(r, \phi_f) + N_2(r, 1/\phi_g) + N_2(r, \phi_g)\}$$

$$+ S(r, \phi_f) + S(r, \phi_g)$$

$$\le 3\{\overline{N}\left(r, \frac{1}{f(qz)-b/a}\right) + \overline{N}\left(r, \frac{1}{g(qz)-b/a}\right)$$
$$+ 2\overline{N}(r, g) + 2\overline{N}(r, f) + \overline{N}(r, 1/g) + \overline{N}(r, 1/f)\}$$
$$+ S(r, f) + S(r, g)$$
$$+ 2\{N_2\left(r, \frac{1}{f(qz)-b/a}\right) + N_2\left(r, \frac{1}{g(qz)-b/a}\right)$$
$$+ 2\overline{N}(r, g) + 2\overline{N}(r, f) + 2\overline{N}(r, 1/f)$$
$$+ 2\overline{N}(r, 1/g) + N(r, f) + N(r, g)\}$$
$$\le 24[T(r, f) + T(r, g)] + S(r, f) + S(r, g) \tag{3.4}$$

Combining (3.2) and (3.3) with the last inequality, we obtain

$$(n-25)[T(r, f) + T(r, g)] \le S(r, f) + S(r, g)$$

which is a contradiction since $n \ge 26$. Lemma 2.5 gives $\phi_f = \phi_g$ or $\phi_f . \phi_g = 1$. Therefore, Theorem 1.1 is thus proved.

Proof of Theorem 1.2. Let ϕ_f and ϕ_g be defined as in (1.1). Suppose that (2.3) holds. By the same arguments as in the proof of Theorem1.1, we note that f and g are entire functions and (3.2)- (3.4) turn out to be

$$(n-1)T(r, f) \le T(r, \phi_f) + S(r, f),$$
$$(n-1)T(r, g) \le T(r, \phi_g) + S(r, g),$$

And

$$T(r, \phi_f) + T(r, \phi_g) \le 12[T(r, f) + T(r, g)] + S(r, f) + S(r, g).$$

Combining the last three inequalities, we get

$$(n-13)[T(r, f) + T(r, g)] \le S(r, f) + S(r, g)$$

which contradicting with $n \ge 14$. Then from Lemma 2.5, we have $\phi_f = \phi_g$ or $\phi_f . \phi_g = 1$.

If $\phi_f = \phi_g$ holds, (1.2) follows by Lemma 2.5.

Next, we assume that $\phi_f . \phi_g = 1$ holds. We know from (1.1) that

$$f^n g^n = (af(qz) - b)(ag(qz) - b) \tag{3.5}$$

We consider two cases in the following.

Case 1. If b=0. By the above equation, we get

$$f^n g^n = a^2 f(qz) g(qz).$$

Let fg=h. If h is not a constant, then by the above equation, we have

$$h^n = a^2 h(qz). \tag{3.6}$$

Thus, from the first main theorem, we obtain

$$\begin{aligned} nT(r, h) &= T(r, h^n) = T(r, ah(qz)) + O(1) \\ &\le T(r, h) + S(r, h) \end{aligned}$$

Since $n \ge 2$, we know that h is a constant, i.e. fg is a constant, so f and g have no zeros. Hence, according to the Decomposition Theorem, we suppose that f(z)=$c_1 e^{p(z)}$, g(z)= $c_2 e^{-p(z)}$, where p(z) is a polynomial. Since that f and g are zero orders, then p(z) is a constant. Therefore, f is a constant, too, which is impossible.

Case 2. If $b \ne 0$. The equation (3.5) gives that

$$N(r,1/f) + N(r,1/g) = \frac{1}{n} N\left(r, \frac{1}{f(qz) - b/a}\right) + \frac{1}{n} N\left(r, \frac{1}{g(qz) - b/a}\right),$$

$$\overline{N}\left(r, \frac{1}{f(qz) - b/a}\right) + \overline{N}\left(r, \frac{1}{g(qz) - b/a}\right) \le 2\overline{N}\left(r, \frac{1}{f}\right) + 2\overline{N}\left(r, \frac{1}{g}\right),$$

and the second main theorem gives that

$$T(r, f(qz)) \le \overline{N}\left(r, \frac{1}{f(qz)}\right) + \overline{N}\left(r, \frac{1}{f(qz) - b/a}\right) + S(r, f)$$

Similarly,

$$T(r, g(qz)) \le \overline{N}\left(r, \frac{1}{g(qz)}\right) + \overline{N}\left(r, \frac{1}{g(qz) - b/a}\right) + S(r, g)$$

The above four inequalities and Lemma 2.2 provide us that

$$\begin{aligned} T(r, f) + T(r, g) &\le \overline{N}\left(r, \frac{1}{f(qz)}\right) + \overline{N}\left(r, \frac{1}{f(qz) - b/a}\right) + S(r, f) \\ &+ \overline{N}\left(r, \frac{1}{g(qz)}\right) + \overline{N}\left(r, \frac{1}{g(qz) - b/a}\right) + S(r, g) \\ &\le 3\left(\overline{N}\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{g}\right)\right) + S(r, f) + S(r, g) \\ &\le \frac{3}{n}\left(N\left(r, \frac{1}{f(z+c) - b/a}\right) + N\left(r, \frac{1}{g(z+c) - b/a}\right)\right) \\ &+ S(r, f) + S(r, g) \\ &\le \frac{3}{n}\left(T(r, f) + T(r, g)\right) + S(r, f) + S(r, g), \end{aligned}$$

contradicting with $n \geq 14$. This completes the proof of Theorem 1.2.

Proof of Theorem 1.3. By the same arguments as in the proof of Theorem 1.1, noting that ϕ_f and ϕ_g share 1 CM, using Lemma 2.6 instead of Lemma 2.5, (3.4) turns out to be

$$\begin{aligned} T(r,\phi_f) &\leq N_2(r,1/\phi_f)+N_2(r,\phi_f)+N_2(r,1/\phi_g)+N_2(r,\phi_g) \\ &+S(r,\phi_f)+S(r,\phi_g) \\ &\leq N_2\left(r,\frac{1}{-af(qz)+b}\right)+2\overline{N}(r,f)+N_2\left(r,\frac{1}{-ag(qz)+b}\right) \\ &+2\overline{N}(r,1/f)+N(r,f)+2\overline{N}(r,g)+N(r,g)+2\overline{N}(r,1/g) \\ &+S(r,f)+S(r,g) \\ &\leq 6[T(r,f)+T(r,g)]+S(r,f)+S(r,g) \end{aligned} \tag{3.7}$$

Similarly,

$$T(r,\phi_g) \leq 6[T(r,f)+T(r,g)]+S(r,f)+S(r,g)$$

Combining the above two inequalities with (3.2), (3.3), we get

$$(n-13)[T(r,f)+T(r,g)] \leq S(r,f)+S(r,g)$$

Which is a contradiction since $n \geq 14$. Lemma 2.6 gives $\phi_f = \phi_g$ or $\phi_f.\phi_g = 1$. Therefore, Theorem 1.3 is thus proved.

Proof of Theorem 1.4. Let ϕ_f and ϕ_g be defined as in (1.1). By the same arguments as in the proof of Theorem 1.1, noting that f and g are entire functions, (3.7) becomes

$$T(r,\phi_f) \leq 3[T(r,f)+T(r,g)]+S(r,f)+S(r,g)$$

We insert the above inequality into (3.2), resulting in

$$(n-1)T(r,f) \leq 3[T(r,f)+T(r,g)]+S(r,f)+S(r,g)$$

The same inequality holds for T(r,g), then

$$(n-7)[T(r,f)+T(r,g)] \leq S(r,f)+S(r,g)$$

which contradicting with $n \geq 8$. Then from Lemma 2.4, we have $\phi_f = \phi_g$ or $\phi_f.\phi_g = 1$, which means that (1.2) or (1.3) holds.

The following proof is similar with that of proof of Theorem 1.2, we omit it here.

4 Conclusion

In this paper, we introduce uniqueness problems of two q-difference polynomials sharing one value and obtain four results. They will be very useful for us to solve this kind functions.

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Competing Interests

Authors have declared that no competing interests exist.

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