British Journal of Mathematics & Computer Science
9(4): 367-379, 2015, Article no.BJMCS.2015.209
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

Reservoir Network Modelling with Alternative Reservoir and Sediment Considerations

M. K. A. Abdulrahman[1*], A. A. Mohd Ismail[2] and Kek Sie Long[3]

[1]*Department of Mathematics, Statistics and Computer Science, Kaduna Polytechnic, Kaduna, Nigeria.*
[2]*Centre for Industrial and Applied Mathematics, Universiti Teknologi Malaysia, Malaysia.*
[3]*Department of Mathematics, Universiti Tun Hussein Onn, Malaysia.*

Article Information

DOI: 10.9734/BJMCS/2015/17200
Editor(s):
(1) Sheng Zhang, Department of Mathematics, Bohai University, Jinzhou, China.
Reviewers:
(1) Anonymous, Libya.
(2) Tenzer Robert, Wuhan University, China.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=1145&id=6&aid=9592

Original Research Article

Received: 02 March 2015
Accepted: 22 April 2015
Published: 04 June 2015

Abstract

Consideration of alternative reservoir in reservoir network for flood control is born out of the need to provide a complete formulation and provide solution to problem associated with sediment inflow. A set of $(m+2)$ coupled nonlinear ODE is formulated for design and management of reservoir network for flood control. At steady state, the normal operating height of each reservoir in the network is determined. Linearizing the nonlinear systems and evaluating it at the obtained normal operating height give a clear and robust approach to determine the stability of the system. This approach to reservoir network modelling with inclusion of alternative reservoir gives a criterion for pump selection, flexible way of reservoir location and complete analysis of the network.

Keywords: Mathematical modelling; reservoir; steady state; nonlinear system; ODE; linearization.

1 Introduction

Mathematical modelling has remained efficient and cost effective way of studying a real life problem [1]. Reservoir network modelling for flood control cannot be an exception in areas of design and location of reservoir for management and control of runoff in this era. The conflicting scenario of scarcity of potable water and flooding requires a huge investment in study of reservoir for water use and flood control. The interplay between the wet and dry season on one hand and effect of global warming on the other hand should therefore be a driving force to invest in sustainable approach to the reservoir management.

Corresponding author: kamilabdulraman@yahoo.com;

The diverse nature of environmental modelling has also made the reservoir management imperative to find a unified approach to its modelling and eventual analysis. The projection of an annual water to be controlled between 9000 km^3 and 14000 km^3 as discussed by [2] makes management of reservoirs in term of retention pond, detention pond and dams to be a necessity. The continuous industrialization and urbanization have also made most catchment to be more impervious which has increased the volume of runoff [3-5]. This increase has made it impossible for a single reservoir to militate against flooding, rather a network of reservoir [6-8]. The new age has also come with sophistication in weather forecast which makes flood forecasting a near accurate but still flood in on the increase. The missing link between the weather forecasting and increase in incident of flood is therefore an effective management of runoff for domestic use and flood control.

The work reported in [9] utilizes the laboratory coupled tank approach to model interrelationship between reservoirs in a network for flood control, while the work in [10] determines the normal operating heights of the reservoirs in the network. The stability of the network reported in [11] is achieved through linearization of the nonlinear system. It was found that for a series connection of reservoirs, the Jacobian matrix associated with the linearized system is a tridiagonal diagonally dominant matrix with negative diagonal element. For a hypothetical connection of reservoirs on the other hand, the corresponding Jacobian matrix is only a diagonally dominant. The former has been proved to be stable [10], while the stability of the later still depends on the matrix entries. The results from [10,9] consider only water due to runoff which is the reason why there is an increase in flood cases, while the sediment consideration in [11] does not include alternative reservoir. The combine effect of increase in rain intensity, short inter-event time and long rain event have shown that the control reservoir (retention pond) may reach flood height over a short period. These are in addition to continuous sediment inflow which reduces the useful capacity of the reservoir. The excess water must be stored in location upland to reduce discharge to the connecting stream thereby preventing downstream from flooding. The model presented in this work therefore introduce alternative reservoir to serve as storage, control of water level and regulate activities in the retention pond.

2 Mathematical Formulation

The mathematical model presented in [9], which is modified for discharge through a weir in [10], is

$$\left.\begin{aligned}
A_j\dot{H}_j &= \begin{cases} 0 & ;\quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ;\quad H_j \le h_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\
A_m\dot{H}_m &= \begin{cases} 0 & ;\quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) - B_m\left(H_m - H_s\right)^{3/2} - A_m f & ;\quad H_m \le h_m^{temp} \end{cases} \quad j = m
\end{aligned}\right\} \tag{1}$$

and sediment consideration in [11] gives

$$A_m\dot{H}_m = \begin{cases} 0 & ;\ H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + T_e K\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)^{\rho} & \\ -B_m H_m^{\ 3/2} - A_m f & ;\ H_m \le h_m^{temp} \end{cases} \tag{2}$$

The sediment term in equation (2) consider the sediment inflow and sediment outflow through a weir. This equation does not give any information in respect of where the sediment flows to and the quantity discharged to the connecting stream. In order to achieve this, sediment transport in the entire catchment is captured from the inflow-outflow from the retention pond. The activities in the detention pond in a close cycle will not affect the formulation since detention pond discharges its entire content after the rain event. Equation (2) is modified on the retention pond to give

$$\left.\begin{array}{l} A_j\dot{H}_j = \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \end{cases} \quad j = 1,2,...,m-1 \\ \\ A_m\dot{H}_m = \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} & \\ -K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} - B_m(H_m - H_s)^{3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \end{array}\right\} \tag{3}$$

where $B_i = c_{d,i}a_i\sqrt{2g}$, $B_m = c_{d,m}L$ and $c_{d,m} = \left(\frac{2}{3}\right)^{3/2}\sqrt{g}$ [12]. The surface area A_j is also related to the area of sub-catchment A_{sc_j} by

$$A_j = \left(\frac{A_{sc_j} I_{sc_j} t_{c_j}}{k_j}\right)^{2/3} \tag{4}$$

where $A_j \le A_{sc_j}$, $k = 1/120$ and runoff coefficient is approximately 1 for complete pervious catchment [9]. However, the complete list of variables and parameters used in this work is presented in Table 1. The difference in the height obtained in equations (3) and (1) gives the effective height of retention pond in the nonlinear form. This height corresponds to the effective volume of the retention pond.

Table 1. Variables and parameters

Parameters	Descriptions
A_j	Surface area of reservoir j (m^2)
A_{sc_j}	Area of sub-catchment j (m^2)
a_j	Area of orifice j (m^2)
A_{A_r}	Surface area of alternative reservoir (m^2)
c_j	Runoff coefficient
$c_{d,j}$	Discharge coefficient of reservoir j
f	Infiltration rate (m^3s^{-1})
g	Acceleration due to gravity (ms^{-2})
H_j	Height of water in reservoir j in nonlinear form (m)
H_j^0	Normal operating height of reservoir j (m)
H_s	Height of water in the stream in nonlinear form (m)
H_s^0	Normal operating height of the stream (m)
H_{A_r}	Height of water in the alternative reservoir in nonlinear form (m)

Table 1 continued……..

$H_{A_r}^0$	Normal operating height of the alternative reservoir (m)
H_m^{temp}	Temporary height of retention pond (m)
$H(s)$	Steady state height (m)
h_j	Height of water in reservoir *j* in linear form (m)
h_s	Height of water in the stream in linear form (m)
h_{A_r}	Height of water in the alternative reservoir in linear form (m)
I	Rain rate (m^3s^{-1})
K_p	Pump constant
L	Length of weir (m)
L_s	Average stream discharge (m^3s^{-1})
l	Characteristic length of the surface of reservoir (m)
m	Number of reservoir in the network
t_c	Time of concentration (s)
T_e	Trap efficiency (%)
V_p	Pump voltage (*Volt*)

3 Network Modification

The network configurations will determine the complexities of the governing equations as the summation sign gives the number of inflow reservoir to the referenced reservoir. In Fig. 1, the series reservoir connection shows the number of reservoirs that is the same level of the reservoir, while in the hypothetical reservoir in
Fig. **2** it shows that the number of reservoirs is greater than the levels of the reservoirs.

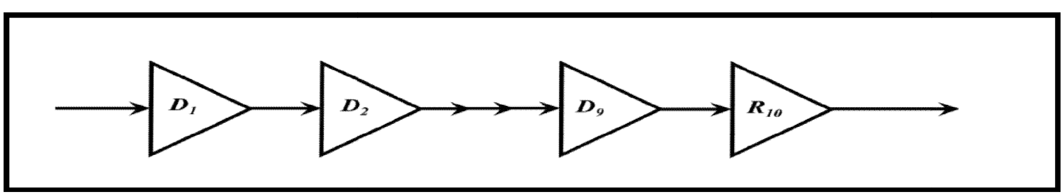

Fig. 1. Series network connection with 10 reservoirs

The continuous flow of water to the stream especially when the inflow is higher than the stream discharges, the height of water in the stream continue to rise beyond its normal operating height. To guard against overflow, diversion of flow from retention pond becomes necessary as the water height gets to the flood level. This diversion cannot be achieved through the free flow any longer as the water has to be pumped to the alternative reservoir. The inflow and outflow of water from the alternative reservoir and the outflow from the retention pond to the stream and alternative reservoir give a complete formulation. The inclusion of alternative reservoir is shown in
Fig. **3**.

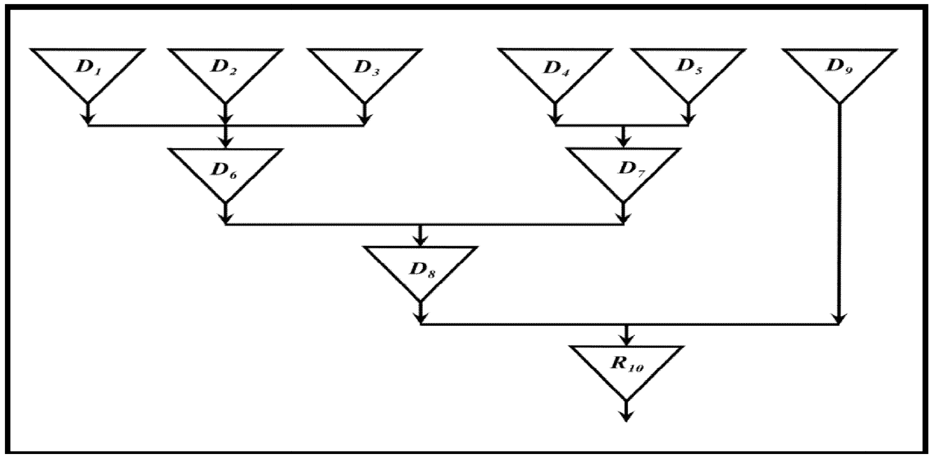

Fig. 2. Hypothetical connection with 10 reservoirs

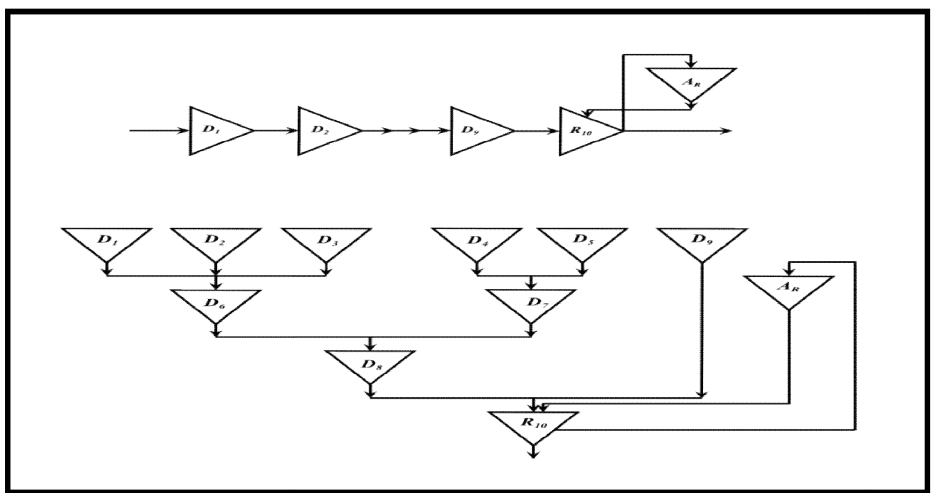

Fig. 3. Complete network in series and hypothetical connection with alternative reservoir

Based on the inclusion of the alternative reservoir in both configurations as shown in Fig. **3**, equation (3) becomes

$$\left.\begin{array}{l}
A_j\dot{H}_j = \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le H_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\
A_m\dot{H}_m = \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} + B_{A_r}\sqrt{H_{A_r} - H_m} & \\ -B_m(H_m - H_s)^{3/2} - A_m f - V_p K_p B_n\sqrt{\left|H_m - H_{Ar}\right|} - K(1 - T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} & ; \quad H_m < H_m^{temp} \end{cases} \\
A_{A_r}\dot{H}_{A_r} = \begin{cases} 0 & ; \quad H_{A_r} > H_{A_r}^0 \\ V_p K_p B_n\sqrt{\left|H_m - H_{Ar}\right|} - B_{A_r}\sqrt{H_{A_r} - H_m} & ; \quad H_{A_r} < H_{A_r}^0 \end{cases} \\
A_s\dot{H}_s = B_m\left(H_m - H_s\right)^{3/2} - L_s \qquad ; \quad H_S < H_S^0
\end{array}\right\} \tag{5}$$

Equation (5) is a full description of the network with alternative reservoir, the stream and sediment inflow consideration in the network. This can be used to study the entire behavior of the network and each of the parameter that contributes to the complete description of the network.

4 Determination of Normal Operating Height

To study the stability of the system represented by equation (5), which is a general equation for reservoir network with sediment consideration and connecting stream. The steady state heights are the points in which stability of the network will be considered. In this context, the steady state heights are called the normal operating heights. Since the network configuration is dependent on the network topology, the hierarchy of the reservoirs in the network becomes necessary. A series network connection has the level of reservoir in the network the same for the number of reservoirs in the network. But, in a network with multiple reservoir inflows, the levels of the reservoirs are less than the number of reservoirs in the network. The hypothetical network in Fig. 3 has 10 reservoirs with 4 level hierarchies. This is a useful when studying the optimization of the network.

The steady state heights of m connected reservoirs are obtained by solving the system of algebraic equations arising from equation (5) when $dH_j/dt = 0 \ \forall j$. Starting with reservoirs $j = 1, 2, ..., m-1$ and considering the first level of reservoirs without inflow from previous reservoir, $i = 0$, equation (5) reduces to

$$H_j(s) = \left(\frac{A_{sc_j} I}{B_j} \right)^2 + H_k(s) \tag{6}$$

For $i \neq 0$, the index of the connected reservoirs changes since the outflow of preceding reservoir is the inflow of the succeeding reservoir, that is, in equation (6), $j := i, \ and \ k := j$. Substituting equation (6) into equation (5) using the new index we have

$$A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{A_{sc_i} I}{B_i} \right)^2 + H_j - H_j} \right) - B_i \sqrt{H_j - H_k} = 0$$

or

$$A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} \left(A_{sc_i} I \right) - B_i \sqrt{H_j - H_k} = 0$$

Where $\sigma_1, \sigma_2, ..., \sigma_r$ count the number of reservoirs that flows into the connected reservoir. The term under the summation sign gives all the runoff $A_{sc_i} I$ for $j = 1, 2, ..., m-1$ which flows into reservoir j in addition to $A_{sc_j} I$. Hence, the normal operating height of any reservoir between 1 to m-1 is

$$H_j(s) = \left(\frac{1}{B_i} \left[A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} A_{sc_i} I \right] \right)^2 + H_k(s) \tag{7}$$

For $j = m$, the runoff from sub-catchment 1 to sub-catchment j is summed up. This makes equation (7) to become

$$H_j(s) = \left(\frac{I}{B_i} \sum_{r=1}^{j} A_{sc_r} \right)^2 + H_k(s) \tag{8}$$

Equation (8) could have been sufficient to determine the normal operating height of the retention pond $j = m$ but it has not taking into account the sediment inflow in the formulation. To do this we ssubstitute equation (8) into equation (5) for $j = m$

$$\begin{aligned} A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) + \\ K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) \right)^{\rho} - \\ B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0 \end{aligned}$$

where $j := m - 1 = i \quad and \quad k = m = j$ and simplifying gives

$$\begin{aligned} A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) + K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) \right)^{\rho} \\ - B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0 \end{aligned}$$

The inner summation sums up all runoff that flows into reservoirs in the referenced reservoir, the outer summation sums up the runoff that flows into the reservoir *m* and the runoff due to the sub-catchment *m* itself. This becomes

$$\begin{aligned} A_{sc_m} I + \sum_{i=1}^{m-1} \left(A_{sc_i} I \right) + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - B_m \left(H_m - H_s \right)^{3/2} \\ - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0 \end{aligned}$$

which can be simplified as

$$x^{\rho} + \frac{1}{K(1-T_e)} x - \frac{AI + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - A_m f}{K(1-T_e)} = 0 \tag{9}$$

where

$$x = B_m \left(H_m - H_s \right)^{3/2} \tag{10}$$

Relying on the value of ρ in [13], we take $\rho = 2$ and equation (9) becomes

$$x^2 + bx - c = 0 \tag{11}$$

where

$$b = \frac{e}{(1-T_e)} \quad \text{and} \; c = \frac{eAI + \left(\sum_{i=1}^{m-1}\left(A_{sc_i} I\right)\right)^{\rho} - eA_m f}{(1-T_e)}$$

Seeking the solution to equation (11), the discriminant must be either $b^2 = 4c$ or $b^2 > 4c$, hence the general solution is

$$x = \frac{-e \pm \sqrt{e^2 - 4(1-T_e)\left(eAI + \left(\sum_{i=1}^{m-1}\left(A_{sc_i} I\right)\right)^{\rho} - eA_m f\right)}}{2(1-T_e)} \tag{12}$$

It can be seen in equation (12) that solution does not exist when trap efficiency is 100%. From equation (10), the normal operating height for the m^{th} reservoir is

$$H_m = \left(\frac{x}{B_m}\right)^{2/3} + H_s \tag{13}$$

For the alternative reservoir on the other hand, we can see that setting equation(5) to zero we have

$$H_{A_r} = H_m + \frac{AI + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(A_{sc_i} I\right)\right]^{\rho} - L_s - A_m f - K(1-T_e)L_s^{\rho}}{V_\rho K_\rho B_n - B_{A_r}} \tag{14}$$

This gives the normal operating height of the alternative reservoir. It should be noted that the denominator of equation (14) will play an important role in pump selection, that is, $V_\rho K_\rho B_n - B_{A_r} > 0$ implies that

$$V_\rho K_\rho > \frac{B_{A_r}}{B_n} \tag{15}$$

This is in addition to appropriate value of H_{A_r}, i.e. as $V_\rho K_\rho B_n - B_{A_r} \to 0,\; H_{A_r} \to \infty$. This criterion can be used to site and select the available location of alternative reservoir. Equations (8), (13) and (14) will be used to get all the normal operating heights of the reservoirs in the network.

5 Determination of Stability of the Network

The normal operating height is the steady state on which the stability of the network relies, linearizing the nonlinear system equation (5) around the normal operating height. This is done as follows;

For $j = 1, 2, ..., m-1$

$$\left.\begin{aligned} \frac{\partial \dot{H}_j}{\partial H_i} &= \sum_{i=\sigma_1}^{\sigma_r}\left(\frac{B_i}{2A_j\sqrt{H_i(s)-H_j(s)}}\right) \\ \frac{\partial \dot{H}_j}{\partial H_j} &= -\frac{\partial \dot{H}_j}{\partial H_i} - \frac{\partial \dot{H}_j}{\partial H_k} \\ \frac{\partial \dot{H}_j}{\partial H_k} &= \frac{B_j}{2A_j\sqrt{H_j(s)-H_k(s)}} \end{aligned}\right\} \tag{16}$$

For $j = m$

$$\left.\begin{aligned} \frac{\partial \dot{H}_m}{\partial H_i} &= \frac{1}{2A_m}\left(\sum_{i=\sigma_1}^{\sigma_r}\frac{B_i}{\sqrt{H_i(s)-H_m(s)}} + \sum_{i=\sigma_1}^{\sigma_r}\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2}\right) \\ \frac{\partial \dot{H}_m}{\partial H_m} &= -\frac{1}{2A_m}\sum_{i=\sigma_1}^{\sigma_r}\frac{B_i}{\sqrt{H_i(s)-H_m(s)}} - \frac{1}{2A_m}\sum_{i=\sigma_1}^{\sigma_r}\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2} \\ &\quad -\frac{1}{2A_m}3B_m\left(H_m(s)-H_s(s)\right)^{1/2} - \frac{1}{2A_m}\frac{B_{A_r}}{\sqrt{H_{A_r}(s)-H_m(s)}} \\ &\quad -\frac{1}{2A_m}\frac{V_pK_pB_m}{\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}} - \frac{1}{2A_m}3\rho K(1-T_e)B_m^{\rho}\left(H_m(s)-H_s(s)\right)^{\frac{3\rho-2}{2}} \\ \frac{\partial \dot{H}_m}{\partial H_s} &= \frac{3B_m}{2A_m}\left(H_m(s)-H_s(s)\right)^{1/2} \\ \frac{\partial \dot{H}_m}{\partial H_{A_r}} &= \frac{1}{2A_m}\left(\frac{B_{A_r}}{\sqrt{H_{A_r}(s)-H_m(s)}} + \frac{V_pK_pB_m}{\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}\right) \end{aligned}\right\} \tag{17}$$

For $j = A_r$,

$$\left.\begin{aligned} \frac{\partial \dot{H}_{A_r}}{\partial H_m} &= \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m}(s)} + \frac{V_pK_pB_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}} \\ \frac{\partial \dot{H}_{A_r}}{\partial H_{A_r}} &= -\frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m}(s)} - \frac{V_pK_pB_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}} \end{aligned}\right\} \tag{18}$$

and for $j = s$

$$\left.\begin{aligned}\frac{\partial \dot{H}_s}{\partial H_m} &= \frac{3}{2A_m} B_m \left(H_m(s) - H_s(s)\right)^{2/3} \\ \frac{\partial \dot{H}_s}{\partial H_s} &= -\frac{3}{2A_m} B_m \left(H_m(s) - H_s(s)\right)^{2/3}\end{aligned}\right\} \tag{19}$$

The associated Jacobian matrix [14-17] obtained by linearizing equation (5) is a combination of equations (16), (17) and (18) which has been shown to be the best linear approximation of the nonlinear system [18,19].

$$\left.\begin{aligned}
\dot{h}_j &= \begin{cases} 0 & ; \quad h_j > h_j^0 \\ \alpha_j h_i - \left(\alpha_j + \beta_j\right) h_j + \beta_j h_k & ; \quad h_j \le h_j^0 \\ j = 1, 2, \ldots, m-1 & \end{cases} \\
\dot{h}_m &= \begin{cases} 0 & ; \quad h_m > h_m^{temp} \\ (\alpha_m + \eta_m) h_i - (\alpha_m + \gamma_m + \theta_m & \\ + \sigma_m + \delta_m + \eta_m) h_m + (\sigma_m + \delta_m) h_{A_r} & \\ + \left(\gamma_m + \theta_m\right) h_s & ; \quad h_m \le h_m^{temp} \end{cases} \\
\dot{h}_{A_r} &= \begin{cases} 0 & ; \quad h_{A_r} > h_{A_r}^{\max} \\ (\sigma_{A_r} + \delta_{A_r}) h_m - (\sigma_{A_r} + \delta_{A_r}) h_{A_r} & ; \quad h_{A_r} \le h_{A_r}^{\max} \end{cases} \\
\dot{h}_s &= \begin{cases} 0 & ; \quad h_m > h_s^{\max} \\ \gamma_s h_m - \gamma_s h_s & ; \quad h_m \le h_s^{\max} \end{cases}
\end{aligned}\right\} \tag{20}$$

Where,

$$\begin{bmatrix}
\alpha_j = \sum_{i=\sigma_1}^{\sigma_r} \frac{B_i}{2A_j \sqrt{H_i^0 - H_j^0}} & ; \ \alpha_m = \sum_{i=\sigma_1}^{\sigma_r} \frac{B_i}{2A_m \sqrt{H_i^0 - H_m^0}} & ; \ \beta_j = \frac{B_j}{2A_j \sqrt{H_j^0 - H_k^0}} \\
\sigma_m = \frac{V_p K_p B_n}{2A_m \sqrt{\left|H_m^0 - H_{A_r}^0\right|}} & ; \ \sigma_{A_r} = \frac{V_p K_p B_n}{2A_{A_r} \sqrt{\left|H_m^0 - H_{A_r}^0\right|}} & ; \ \gamma_s = \frac{3B_m}{2A_s}\left(H_m^0 - H_s\right)^{1/2} \\
\delta_m = \frac{B_{A_r}}{2A_m \sqrt{H_{A_r}^0 - H_m^0}} & ; \ \delta_{A_r} = \frac{B_{A_r}}{2A_{A_r} \sqrt{H_{A_r}^0 - H_m^0}} & ; \ \eta_m = \frac{K \rho B_j^{\rho} \left(\sqrt{H_j^0 - H_k^0}\right)^{\rho - 2}}{2A_m} \\
\gamma_m = \frac{3B_m}{2A_m}\left(H_m^0 - H_s\right)^{1/2} & ; \ \theta_m = \frac{3B_m^{\rho} K(1 - T_e)}{2A_m}\left(H_m^0 - H_s^0\right)^{\frac{3\rho - 2}{2}} &
\end{bmatrix} \tag{21}$$

The coefficient matrix associated with equation (20) will produce $(m+2)\times(m+2)$ matrix with the coefficient given in equation (21) and depend on equations (8), (13) and (14). The Jacobian matrix is then used to test for stability of the network i.e. a stable network in this case shows that the normal operating heights obtained will guarantee free flow of water in the network. A series connection will produce a Jacobian matrix that is diagonally dominant tridiagonal matrix with negative entries. This additional property of tridiagonal matrix has been shown in [11] to be negative definite for series connection. This is an extension of the work reported in [20]. However, this cannot be said of hypothetical network which the Jacobian matrix is not tridiagonal.

6 Discussion of Result

The extension of the model and results presented in [9,11] give rise to equation (5) which includes the introduction of alternative reservoir and sediment inflow in the network. These considerations make the determination of normal operating height lead to the quadratic equation (11). This is as a result of taking $\rho = 2$ which satisfy $\rho \in [2,3]$ in [21,13]. The result equation (14) is unique due to flexibility it offers for the selection of the location of the alternative reservoirs in the face of other reservoir location criterion. Moreover, the criterion equation (15) is principally useful in selection of appropriate pump specification. Hence, the stability of the network is dependent on the catchment parameters as well as the sediment inflow which affect the normal operating height of the retention pond. A series network has been shown to produce a diagonally dominant tridiagonal with negative diagonal entries to be negative definite as a criteria for the stability of the network. This cannot be said of hypothetical network since the network configuration will determine the Jacobian matrix. The only information in respect of Jacobian matrix of hypothetical network is negative diagonal diagonally dominant matrix which either similarity transformation or other conversion can ascertain its stability properties.

7 Conclusion

A complete model for reservoir network formulation is achieved through system of nonlinear ODE. This model has set the pace for several studies ranging from stability, control and optimization of the network for flood management. Specifically, in this work, since $\rho \in [2,3]$, there is a need trying $\rho = 3$ which will produce a third order polynomial where appropriate criteria will be developed for a feasible solution to obtain the normal operating heights. Effort are on to determine the constants ρ and K from the stability study of the catchment using inflow characteristics. The result is catchment dependent and must be between $\rho = 2$ and $\rho = 3$ apart from this however, different case studies will eventually give a clear insight into most appropriate choice of ρ .

Acknowledgements

The authors will like to acknowledge the contribution of the Centre for Industrial and Applied Mathematics, University Teknologi, Malaysia for their support towards this research. The first author will also like to acknowledge the management of World Merit Consultancy Limited for their support in broadening research opportunity in reservoir network modelling.

Authors' Contributions

This work was carried out in collaboration between all authors. Author MKAA designed the study, performed the model formulation and result analysis, and wrote the first draft of the manuscript. Author AAMI supervised provided literatures and managed the entire study. Author KSL coordinated and revised the manuscript. All authors read and approved the final manuscript.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Catherine MB. What is modelling and simulation? Principles of modeling and simulation: A multidiscplinary approach, ed. John AS, Catherine MB. Hoboken, New Jersey: John Wiley & Sons, Inc; 2009.

[2] Ahmed MS, Saghyroon E, Sherif ME, Muna M, Semunesh G. Assessment of the current state of the nile basin reservoir sedimentation problems, in Nile Basin Capacity Building Network 'NBCBN'. River Morphology Research Cluster, Ahmed S, Gerrit K, Editors; 2005.

[3] Yiping G, Barry JA. Hydrologic analysis of urban catchments with event-based probabilistic models 1. Runoff volume. Water Resources Research. 1998;34(12):3421–3431.

[4] Yiping G, Barry JA. Hydrologic analysis of urban catchments with event-based probabilistic models 2. Peak discharge rate. Water Resources Research. 1998;34(12):3433–3443.

[5] Edward RB, Maryam K, Yiping H. Effects of impervious area estimation method on the simulated peak discharges. Journal of Hydraulic Engineering. 2009;14(4):388-398.

[6] John WL. Optimal operation of multireservoir systems: State-of-the-art review. Journal of Water Resources Planning and Management. 2004;130(2).

[7] Quentin BT, Larry WM. Optimizing retention basin networks. Journal of Water Resources Planning and Management. 2008;134(5):432–439.

[8] Us Army Corps of Engineers. Reservoir sedimentation. Flood & Coastal Storm Damage Reduction R&D Program; 2010.

[9] Abdulrahman MKA, Mohd Ismail AA, Kek SL. Modelling of reservoir network for flood control using nonlinear reservoir routing model. International Journal of Engineering Science and Technology (IJEST). 2012;4(7):3295-3303.

[10] Abdulrahman MKA, Moh'd Ismail AA, Kek SL. Determination of normal operating heights of reservoirs in a network using non-linear reservoir method. British Journal Of Applied Science & Technology. 2012;3(1):34-47.

[11] Abdulrahman MKA, Mohd Ismail AA, Sie LK. Effects of sediment inflow on reservoir network for flood control. International Journal of Mathematics and Applied Statistics. 2013;45(15):103-110.

[12] John EJ. Introduction to hydraulic and hydrology with application stormwater management. 2nd ed. Delmar Thompson Learning; 1997.

[13] Vaibhav G, Jothiprakash V. Estimation of useful life of a reservoir using sediment trap efficiency. Journal of Spatial Hydrology. 2008;8(2):1-14.

[14] Zoran G. Linear dynamic systems and signals. Prentice Hall; 2003.

[15] Marsden JE, Sirovich L, Golubitsky M, Jäger W, John F, eds. Differential equations and their applications: An introduction to applied mathematics. F ourth ed. Texts in applied mathematics 11, ed. Braun M; 1992. Springer.

[16] Tin-Zhu H. A note on generalired diagonally dominant matrices. Linear Algebra And Its Applications. 1995;237-240.

[17] Yi-Ming G, Xiao-Hui W. Criteria for generalized diagonally dominant matrices and m-matrices II. Linear Algebra And Its Applications. 1996;248:339-353.

[18] Vidyasagar M. Nonlinear system analysis. Englewood Cliff, New Jersey: Prentice-Hall; 1993.

[19] Hassan KK. Nonlinear systems. Upper Saddle: Prentice Hall; 2002.

[20] Andelic M, Dafonseca CM. Sufficient conditions for positive definiteness of tridiagonal matrices revisited. Positivity. 2011;15(1):155-159.

[21] Newton DOC, Wellington CL. Evaluation of the useful life of a reservoir on the river manso, mato grosso state, brazil: A case study. The Hydrological Basis for Water Resources Management (Proceedings of the Beijing Symposium). 1990;197:14.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=1145&id=6&aid=9592