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Properties of Annihilators in Lattice Wajsberg Algebras

A. Ibrahim[1] and C. Shajitha Begum[2*]

[1]P. G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai-622 001, Tamilnadu, India.

[2]Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore-641 022, Tamilnadu, India.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, we introduce the notion of the annihilator in lattice Wajsberg algebra and investigate some related properties of it. We show that the annihilator is a *WI*-ideal. In further note, we discuss relationship between the annihilator and a *WI*-ideal in the article. Moreover, we investigate some properties of the lattice implication homomorphism image of annihilators, and also give the necessary and sufficient condition of the lattice implication homomorphism image, and we obtain lattice implication homomorphism and isomorphism inverse images of annihilators in lattice from Wajsberg algebras.

Keywords: Wajsberg algebra; Lattice Wajsberg algebra; WI-ideal; annihilator; homomorphism; isomorphism.

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**Corresponding author: E-mail: nousheen2407@gmail.com;*
E-mail: ibrahimaadhil@yahoo.com;

1 Introduction

Non-classical logic has become a considerable formal tool for computer science and artificial intelligence to deal with fuzzy information and uncertain information. Many-valued logic, a great extension and development of classical logic, has always been a crucial direction in non-classical logic. In order to research the logical system whose propositional value is given in a lattice. Simge Öztunç, Ali Mutlu, and Necdet Bildik [1] considered the set of lattice points in Euclindean spaces $\mathbb{Z}^n$, and the digital image by using adjacency relation in the subset of $\mathbb{Z}^n$. Further, they [2] constructed the group structure in digital images and defined the simplicial groups in digital images. Yang Xu [3] proposed the concept of lattice implication algebras. The properties of lattice implication algebras were studied in [3] and [4]. Mordchaj Wajsberg [5] proposed the concept of Wajsberg algebras in 1935, to show that the valued $\aleph_0$ Lukariewicz propositional calculus was complete with respect to axiomatic conjectured by Lukariewicz itself. Rose and Rosser [6] published the proof of Wajsberg algebra in 1958. Font, Rodriguez and Torrens [7], introduced the notion of lattice Wajsberg algebras in 1984, which is an algebraic structure that is established by combining a lattice and Wajsberg algebra, and discussed some of their properties. For the general development of lattice Wajsberg algebras, filter theory plays an important role. They [7], introduced the notion of implicative filters in a lattice Wajsberg algebras, and investigated their properties. Basheer Ahamed and Ibrahim [8, 9], introduced the definitions of fuzzy implicative and anti fuzzy implicative filters of lattice Wajsberg algebras and obtained some properties with illustrations. Theory of ideals is another important development of lattice Wajsberg algebras. The authors [10] recently introduced the notion of Wajsberg implicative ideal (*WI*-ideal) of lattice Wajsberg algebra and derived some properties. The concept of an annihilator in lattice was introduced by Mandelker [11] as a generalization of the concept of a pseudo complement. Davey and Nieminen [12] introduced the notion of annihilators in modular lattices and discussed some properties.

The present paper is organized as follows: In Section 2, we review some basic definitions and results about lattice Wajsberg algebras. In Section 3, we introduce the notion of the annihilators and obtain some characteristics of annihilators in lattice Wajsberg algebras. Then we show that the annihilator is a *WI*-ideal, and discuss the relationships between the annihilator and a *WI*-ideal, between the lattice implication homomorphism image of annihilator and the annihilator of lattice implication homomorphism image of lattice Wajsberg algebras, respectively. Moreover, we investigate some characteristics, necessary and sufficient conditions of lattice implication homomorphism inverse image of annihilator. Finally, we discuss the lattice implication isomorphism inverse image of annihilator of lattice Wajsberg algebras.

2 Preliminaries

In this section, we recall some basic definitions and their properties which are helpful to develop our main results.

Definition 2.1. [7] An algebra (A, $\rightarrow$, *****, 1) with quasi complement "*" and a binary operation "$\rightarrow$" is called a Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \rightarrow x = x$

(ii) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$

(iii) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(iv) $(x^* \rightarrow y^*) \rightarrow (y \rightarrow x) = 1$.

Proposition 2.2. [7] A Wajsberg algebra (A, $\to$, *****, 1) satisfies the following axioms for all $x, y, z \in A$,

(i) $x \to x = 1$

(ii) If $(x \to y) = (y \to x) = 1$ then $x = y$

(iii) $x \to 1 = 1$

(iv) $(x \to (y \to x)) = 1$

(v) If $(x \to y) = (y \to z) = 1$ then $x \to z = 1$

(vi) $(x \to y) \to ((z \to x) \to (z \to y)) = 1$

(vii) $x \to (y \to z) = y \to (x \to z)$

(viii) $x \to 0 = x \to 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $(x^* \to y^*) = y \to x$.

Definition 2.3. [7] Let (A, $\to$, *****, 1) be a Wajsberg algebra. Then the partial ordering "$\le$" on A is defined by $x \le y$ if and only if $x \to y = 1$ for all $x, y \in A$.

Definition 2.4. [7] A Wajsberg algebra (A, $\to$, *****, 1) is called a lattice Wajsberg algebra if it satisfies the following axioms for all $x, y \in A$,

(i) $(x \vee y) = (x \to y) \to y$

(ii) $(x \wedge y) = ((x^* \to y^*) \to y^*)^*$.

Note. From the above definition, (A, $\vee$, $\wedge$, *****, 0, 1) is a lattice Wajsberg algebra with lower bound 0 and upper bound 1.

Proposition2.5. [7] A lattice Wajsberg algebra (A, $\to$, *****, 1) satisfies the following axioms for all $x, y, z \in A$,

(i) If $x \le y$ then $x \to z \ge y \to z$ and $z \to x \le z \to y$

(ii) $x \le y \to z$ if and only if $y \le x \to z$

(iii) $(x \vee y)^* = (x^* \wedge y^*)$

(iv) $(x \wedge y)^* = (x^* \vee y^*)$

(v) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

(vi) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$

(vii) $(x \to y) \vee (y \to x) = 1$

(viii) $x \to (y \vee z) = (x \to y) \vee (x \to z)$

(ix) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$

(x) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xi) $(x \wedge y) \to z = (x \to y) \to (x \to z)$.

Definition 2.6. [7] Let L be a lattice. An ideal I of L is a nonempty subset of L is called a lattice ideal if it satisfies the following axioms for all $x, y \in I$,

(i) $x \in I$, $y \in L$ and $y \le x$ imply $y \in I$

(ii) $x, y \in I$ implies $x \vee y \in I$.

Definition 2.7. [10] Let A be a lattice Wajsberg algebra. Let I be a nonempty subset of A is called a *WI*-ideal of lattice Wajsberg algebra A if it satisfies the following axioms for all $x, y \in A$,

(i) $0 \in I$

(ii) $(x \to y)^* \in I$ and $y \in I$ imply $x \in I$

Proposition 2.8. [10] If R is a non-empty family if *WI*-ideals of a lattice Wajsberg algebra A. Then $I = \bigcap R$ is also a *WI*-ideal of A.

Let I be a subset of a lattice Wajsberg algebra A. Then the least *WI*-ideal containing I is called a *WI*-ideal generated by I written $\langle I \rangle$ of I and is also called the *WI*-ideal generated by I. If $I = \{a\}$, we write $\langle I \rangle = \langle a \rangle$.

Definition 2.9. [10] Let A_1 and A_2 be lattice Wajsberg algebras, $f : A_1 \to A_2$ be a mapping from A_1 to A_2 if for any $x, y \in A_1$, $f(x \to y) = f(x) \to f(y)$ holds, then f is called an implication homomorphism from A_1 to A_2. If f is an implication homomorphism and satisfies,

(i) $f(x \vee y) = f(x) \vee f(y)$

(ii) $f(x \wedge y) = f(x) \wedge f(y)$

(iii) $f(x^*) = (f(x))^*$ for all $x, y \in A_1$, then f is called lattice implication homomorphism from A_1 to A_2.

3 Main Results

3.1 Properties of annihilators

In this section, we define an annihilator of lattice Wajsberg algebra and investigate some related properties with illustrations.

Definition 3.1.1. Let S be a non-empty subset of a lattice Wajsberg algebra A if $S^{\perp} = \{x \in A / x \wedge a = 0 \text{ for all } a \in S\}$, then $S^{\perp}$ is called the annihilator of S.

Example 3.1.2. Let $A = \{0, a, b, c, d, 1\}$ be a set with Fig. 1 as a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on A as in Table 1 and Table 2.

Table 1. Complement

x	x^*
0	1
a	c
b	d
c	a
d	b
1	0

Table 2. Implication

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

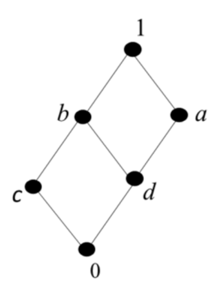

Fig. 1. Lattice diagram

Define $\vee$ and $\wedge$ operations on A as follows,

$(x \vee y) = (x \to y) \to y$,

$(x \wedge y) = ((x^* \to y^*) \to y^*)^*$ for all $x, y \in A$.

Then, $(A, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra.

Let $S = \{0, c\}$ it is easy to check that $S^\perp = \{0, a, d\}$ is the annihilator of S.

Let $T = \{0, a\}$ then $T^\perp = \{0, c\}$ is the annihilator of T.

Note. Obviously, the annihilator of $\{0\}$ is A.

Proposition 3.1.3. Let A be a lattice Wajsberg algebra, S be a non-empty subset of A. If $S^\perp$ is the annihilator of S, then $a \in S$, $x \to a = x^*$ if and only if $x \in S^\perp$ for any $x \in A$.

Proof. Let $a \in S$, $x \to a = x^*$ for all $x \in A$.

Then, we have $(x \to a) \to x^* = 1$

$(a^* \to x^*) \to x^* = 1$

$(a^* \vee x^*) = 1$

$(x \wedge a) = 0$

Thus, $x \in S^\perp$.

Conversely, $x \in S^\perp$, then $x \wedge a = 0$ for all $a \in S$.

It follows that $(a \wedge x)^* = a^* \vee x^* = (a^* \to x^*) \to x^* = 1$.

Then, we have $a^* \to x^* \le x^*$, $x^* \le a^* \to x^*$ is trivial.

Thus, $a^* \to x^* = x^*$, and so $x \to b = x^*$.

Proposition 3.1.4. Let A be a lattice Wajsberg algebra and $a \in A$. If $\{a\}^\perp$ is the annihilator of $\{a\}$, then for any $x \in \{a\}^\perp$; $a \le x^*$, and $x \le a^*$.

Proof. Let $\{a\}^\perp$ is the annihilator of $\{a\}$. Then, for any $x \in \{a\}^\perp$, by the proposition 3.1.3, we have $x \to a = x^*$ $x \to a = x^*$. Since $x^* \vee a \to (x \to a) = (x^* \to (x \to a)) \wedge (a \to (x \to a)$

$= (a^* \to (x^* \to x^*) \wedge (x \to (a \to a)) = 1.$

We have $x^* \vee a \le x \to a$

Therefore, $x^* \vee a = x^*$ and so $a \le x^*$ and $x \le a^*$.

Proposition 3.1.5. Let A be a lattice Wajsberg algebra and $a, b \in A$. If $a \le b$, then $\{b\}^\perp \subseteq \{a\}^\perp$.

Proof. For all $x \in \{b\}^\perp$, we have $(x \wedge a) \le (x \wedge b) = 0$, so $x \wedge a = 0$, and so, we have $x \in \{a\}^\perp$. Therefore, we get $\{b\}^\perp \subseteq \{a\}^\perp$.

Proposition 3.1.6. Let A be a lattice Wajsberg algebra. If S is a non-empty subset of A, and $S^{\perp}$ is the annihilator of S, then $S^{\perp}$ is a *WI*- ideal of A.

Proof. Let $S^{\perp}$ is the annihilator of S. Then $0 \in S^{\perp}$ (by the definition 3.1.1)

Assume that $(x \to y)^* \in S^{\perp}$, $y \in S^{\perp}$ for all $x, y \in A$.

Then, we have $y \to a = y^*$ and $(x \to y)^* \to a = x \to y$, by the proposition 3.1.3.

It follows that $x^* = 1 \to x^*$

$$
\begin{aligned}
&= ((y \to a) \to y^*) \to x^* \\
&= ((a^* \to y^*) \to y^*) \to x^* \\
&= ((a^* \vee y^*) \to x^* \\
&= (a^* \to x^*) \wedge (y^* \to x^*) \\
&= (a^* \to x^*) \wedge (x \to y) \\
&= (a^* \to x^*) \wedge ((x \to y)^* \to a) \\
&= (a^* \to x^*) \wedge (a^* \to (y^* \to x^*)) \\
&= (a^* \to x^*) = (x \to a).
\end{aligned}
$$

Thus, $x \in S^{\perp}$ by the proposition 3.1.3. Hence, $S^{\perp}$ is a *WI*- ideal of A.

Proposition 3.1.7. Let A be lattice Wajsberg algebra, if S and T are non-empty subsets of A. Then the following are hold.

(i) If $S \subseteq T$ then $T^{\perp} \subseteq S^{\perp}$;

(ii) $S^{\perp} \subseteq S^{\perp\perp}$;

(iii) $S^{\perp} = S^{\perp\perp\perp}$;

(iv) $(S \cup T)^{\perp} = S^{\perp} \cap T^{\perp}$;

(v) $(S^{\perp} \cap T^{\perp}) \subseteq (S \cap T)^{\perp}$.

Proof. (i) Let $S \subseteq T$. For any $x \in T^{\perp}$, then $x \wedge a = 0$ for all $a \in T$, and so $x \wedge b = 0$ for all $b \in S$.

Thus, we have $x \in S^{\perp}$. Hence $T^{\perp} \subseteq S^{\perp}$.

(ii) For any $a \in S$, $x \in S^{\perp}$, $x \wedge a = 0$, then, we get $a \in S^{\perp\perp}$. Thus, $S^{\perp} \subseteq S^{\perp\perp}$.

(iii) From (ii), we have $S^{\perp} \subseteq S^{\perp\perp\perp}$ and $S \subseteq S^{\perp\perp}$, and $S^{\perp\perp\perp} \subseteq S^{\perp}$, by (i).

Therefore, $S^{\perp} = S^{\perp\perp\perp}$.

(iv) Since $S \subseteq S \cup T$ and $T \subseteq S \cup T$, by (i), $(S \cup T)^{\perp} \subseteq S^{\perp}$ and $(S \cup T)^{\perp} \subseteq T^{\perp}$.

Then, we have $(S \cup T)^{\perp} \subseteq S^{\perp} \cap T^{\perp}$. (1)

For any $x \in S^{\perp} \cap T^{\perp}$. Then $x \in S^{\perp}$ and $x \in T^{\perp}$.

Thus, we have $x \wedge a = 0$ and $x \wedge b = 0$ for all $a \in S$, $b \in T$.

Then, $x \wedge (a \vee b) = 0$, and so $x \in (S \cup T)^{\perp}$. Hence $S^{\perp} \cap T^{\perp} \subseteq (S \cup T)^{\perp}$. (2)

From (1) and (2), we get $(S \cup T)^{\perp} = S^{\perp} \cap T^{\perp}$.

(v) From (iii), we have $(S\cup T)^{\perp}=S^{\perp}\cap T^{\perp}$, since $S\cap T\subseteq S\cup T$.

From (i), we get $(S\cup T)^{\perp}\subseteq (S\cap T)^{\perp}$, by (iv) $(S^{\perp}\cap T^{\perp})\subseteq (S\cap T)^{\perp}$.

Corollary 3.1.8. Let A be a lattice Wajsberg algebra. If S is a non-empty subset of A, then $S^{\perp}=\bigcap\limits_{a\in S}\{a\}^{\perp}$.

Proof. From (iv) of proposition 3.1.7 and $S=\bigcup\limits_{a\in S}\{a\}$, we get $S^{\perp}=\left(\bigcup\limits_{a\in S}\{a\}\right)^{\perp}=\bigcap\limits_{a\in S}\{a\}^{\perp}$.

Proposition 3.1.9. Let A be a lattice Wajsberg algebra. If S is a non-empty subset of A and $\langle S\rangle=\langle S\rangle^{\perp\perp}$, then $\langle S\rangle=S^{\perp\perp}$.

Proof. It is clear that $S\subseteq\langle S\rangle$, then from (i) of proposition 3.1.7, we have $\langle S\rangle^{\perp}\subseteq S^{\perp}$ and $S^{\perp\perp}\subseteq\langle S\rangle^{\perp\perp}$, so $S^{\perp\perp}\subseteq\langle S\rangle$. (3)

Since $\langle S\rangle\subseteq\langle S\rangle^{\perp\perp}$, $S^{\perp\perp}\subseteq\langle S\rangle$. From (ii) of proposition 3.1.7, we have $S\subseteq S^{\perp\perp}$, and from the proposition 3.1.6, $S^{\perp\perp}$ is a *WI*- ideal of A. Thus, we have $\langle S\rangle\subseteq S^{\perp\perp}$. (4)

From (3) and (4), we get $\langle S\rangle=S^{\perp\perp}$.

Note. From the proposition 3.1.9, $\langle S\rangle=S^{\perp\perp}$ is called the generated *WI*-ideal.

Proposition 3.1.10. Let A be a lattice Wajsberg algebra. If S and T are non-empty subset of A, then $\left\langle S^{\perp}\cup T^{\perp}\right\rangle\subseteq(S\cap T)^{\perp}$.

Proof. We know that $S\cap T\subseteq S$ and $S\cap T\subseteq T$, then from (i) of proposition 3.1.7 we have $S^{\perp}\subseteq(S\cap T)^{\perp}$ and $T^{\perp}\subseteq(S\cap T)^{\perp}$. Thus, $S^{\perp}\cup T^{\perp}\subseteq(S\cap T)^{\perp}$. By the proposition 3.1.6, $(S\cap T)^{\perp}$ is a *WI*-ideal of A. Hence, we have $\left\langle S^{\perp}\cup T^{\perp}\right\rangle\subseteq(S\cap T)^{\perp}$.

Proposition 3.1.11. Let A be a lattice Wajsberg algebra. If S is a *WI*-ideal of A, then $S\cap S^{\perp}=\{0\}$.

Proof. Obviously, $0\in S\cap S^{\perp}$, and $\{0\}\subseteq S\cap S^{\perp}$ (5)

If for any $x\in S\cap S^{\perp}$, then $x\in S$ and $x\in S^{\perp}$. From the definition 3.1.1, we have $x=x\wedge x=0$. Thus, $x\in\{0\}$, and so $S\cap S^{\perp}\subseteq\{0\}$. (6)

From (5) and (6), we have $S\cap S^{\perp}=\{0\}$.

Proposition 3.1.12. Let A be a lattice Wajsberg algebra. If S and T are *WI*-ideal of A, then $S\cap T=\{0\}$ if and only if $S\subseteq T^{\perp}$.

Proof. Let $S\cap T=\{0\}$ then for any $x\in S$, $a\in T$ and $x\wedge a=0$ or $x\wedge a\neq 0\in S\cap T$. Which is a contradiction to $S\cap T=\{0\}$. Thus $x\in T^{\perp}$. From the definition 3.1.1, we have $S\subseteq T^{\perp}$. Conversely, if

$S \subseteq T^{\perp}$, we have $S \cap T \subseteq T^{\perp} \cap T$. From the proposition 3.1.11, we have $T \cap T^{\perp} = \{0\}$. Thus, $S \cap T = \{0\}$.

Corollary 3.1.13. Let A be a lattice Wajsberg algebra, S and T be *WI*-ideals of A. If $T = T^{\perp\,\perp}$. Then $S \subseteq T$ if and only if $S \cap T^{\perp} = \{0\}$.

Proof. Let $S \subseteq T = T^{\perp\,\perp}$. By the proposition 3.1.6, we know that $T^{\perp}$ is a *WI*-ideal of *A*. Thus, from the proposition 3.1.12, we have $S \cap T^{\perp} = \{0\}$. Conversely, if $S \cap T^{\perp} = \{0\}$. Then, from the proposition 3.1.12, we have $S \subseteq T = T^{\perp\,\perp}$.

3.2 Homomorphism image of annihilators

In this section, we discuss the relation between the lattice implication homomorphism image of annihilator and an annihilator of lattice implication homomorphism image.

Proposition 3.2.1. Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a lattice implication homomorphism. If *S* is a non-empty subset of A_1, then $f(S^{\perp}) \subseteq \left(f(S)\right)^{\perp}$.

Proof. For all $x \in f(S^{\perp})$, there is a $y \in S^{\perp}$, such that $x = f(y)$. For all $z \in f(S)$, there is a $t \in S$, such that $z = f(t)$. So, we have $x \wedge z = f(y) \wedge f(t) = f(y \wedge t) = f(0) = 0$, thus, $x \in \left(f(S)\right)^{\perp}$. Hence, we have $f(S^{\perp}) \subseteq \left(f(S)\right)^{\perp}$.

The next example shows the following: Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a lattice implication homomorphism, if *S* is a non-empty subset of A_1. Then $f(S^{\perp})$ may not be the annihilator of a subset of *T*.

Example 3.2.2. Let $A_1 = \{0, a, b, 1\}$ be a set, where $0 \le a, b \le 1$ is a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on A_1 as in Table 3 and Table 4.

Table 3. Complement

x	x^*
0	1
a	b
b	a
1	0

Table 4. Implication

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

Define $\vee$ and $\wedge$ operations on *A* as follows:
$(x \vee y) = \max\{x, y\}$
$(x \wedge y) = \min\{x, y\}$ for all $x, y \in A_1$.
Then, $(A_1, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra.

Let $A_2 = \{0, 0.5, 1\}$, such that $0 \le 0.5 \le 1$, we have Tables 5 and 6 as follows:

Table 5. Complement

x	x^*
0	1
0.5	0.5
1	0

Table 6. Implication

$\to$	0	0.5	1
0	1	1	1
0.5	0.5	1	1
1	0	0.5	1

Then $(A_2, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra.

Let $f(1) = f(a) = 1$, $f(0) = f(b) = 0$, then $f : A_1 \to A_2$ is a homomorphism. Let $S = \{b\}$, then $S^{\perp} = \{0, a\}$, $f(S^{\perp}) = \{0, 1\}$, clearly $\{0, 1\}$ is not down set, so there is no $T \subseteq A_2$, such that $f(S^{\perp}) = T^{\perp}$.

Proposition 3.2.3. Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a onto lattice implication homomorphism. If T is a non-empty subset of A_2, then $(f^{-1}(T))^{\perp} \subseteq f^{-1}(T^{\perp})$.

Proof. For all $x \in (f^{-1}(T))^{\perp}$ and for all $t \in T$, there is a $s \in A_1$, such that $t = f(s)$, so $x \wedge s = 0$, then $f(x) \wedge t = f(x) \wedge f(s) = f(x \wedge s) = f(0) = 0$. Therefore, we have $f(x) \in T^{\perp}$, which implies that $x \in f^{-1}(T^{\perp})$. Hence, we have $(f^{-1}(T))^{\perp} \subseteq f^{-1}(T^{\perp})$.

Proposition 3.2.4. Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a lattice implication homomorphism. If S is a non-empty subset of A_1, then $f(S^{\perp}) = \left(f(S)\right)^{\perp}$ if and only if $(f(S^{\perp}))^{\perp\perp} = f(S^{\perp})$ and $(f(S))^{\perp} \cap (f(S^{\perp}))^{\perp} = \{0\}$.

Proof. Let $f(S^{\perp}) = \left(f(S)\right)^{\perp}$, then, we have $(f(S^{\perp}))^{\perp\perp} = \left(f(S)\right)^{\perp\perp\perp} = (f(S))^{\perp} = f(S^{\perp})$, $(f(S))^{\perp} \cap (f(S^{\perp}))^{\perp} = (f(S))^{\perp} \cap (f(S))^{\perp\perp} = \{0\}$. Conversely, from the proposition 3.2.1, we have $f(S^{\perp}) \subseteq \left(f(S)\right)^{\perp}$. Next we prove that $\left(f(S)\right)^{\perp} \subseteq f(S^{\perp})$, since $(f(S))^{\perp} \cap (f(S^{\perp}))^{\perp} = \{0\}$, we have $(f(S))^{\perp} \subseteq (f(S^{\perp}))^{\perp\perp} = f(S^{\perp})$. Hence, we get $f(S^{\perp}) = \left(f(S)\right)^{\perp}$.

Proposition 3.2.5. Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a onto lattice implication homomorphism. If T is a non-empty subset of A_2, then $(f^{-1}(T))^{\perp} = f^{-1}(T^{\perp})$ if and only if $f^{-1}(T^{\perp}) \cap (f^{-1}(T))^{\perp\perp} = \{0\}$.

Proof. Let $(f^{-1}(T))^{\perp} = f^{-1}(T^{\perp})$. Then, $f^{-1}(T^{\perp}) \cap (f^{-1}(T))^{\perp\perp} = (f^{-1}(T))^{\perp} \cap (f^{-1}(T))^{\perp\perp} = \{0\}$. Conversely, if $x \in f^{-1}(T^{\perp})$, $y \in A_1$, such that $y \le x$, then $f(y) \le f(x)$, since $f(x) \in T^{\perp}$, we have $f(y) \in T^{\perp}$, so $y \in f^{-1}(T^{\perp})$, and so we have $f^{-1}(T^{\perp})$ is a down set. Since $f^{-1}(T^{\perp}) \cap (f^{-1}(T))^{\perp\perp} = \{0\}$, we have $x \in (f^{-1}(T))^{\perp}$. From the proposition 3.2.3, we have $(f^{-1}(T)) \subseteq f^{-1}(T^{\perp})$. Hence, $(f^{-1}(T))^{\perp} = f^{-1}(T^{\perp})$.

Proposition 3.2.6. Let A_1, A_2 be two lattice Wajsberg algebras, $f : A_1 \to A_2$ be a lattice implication isomorphism. If S and T are non-empty subsets of A_1 and A_2 respectively, then the following hold: (i) $f(S^{\perp}) = (f(S))^{\perp}$ and (ii) $(f^{-1}(T))^{\perp} = (f^{-1}(T^{\perp}))$.

Proof. (i) From the proposition 3.2.1, we have $f(S^{\perp}) \subseteq \left(f(S)\right)^{\perp}$, now we prove that $\left(f(S)\right)^{\perp} \subseteq f(S^{\perp})$. For $y \in (f(S))^{\perp}$, since f is onto, there is a $x \in A_1$, such that $f(x) = y$. For any $s \in S$, we have $f(s) \in f(S)$, so $f(x \wedge s) = f(x) \wedge f(s) = y \wedge f(s) = 0$. Since f is one-to-one, we have $x \wedge s = 0$, so $x \in S^{\perp}$, that is $y \in f(S^{\perp})$. Therefore, $\left(f(S)\right)^{\perp} \subseteq f(S^{\perp})$. Hence, we have $f(S^{\perp}) = (f(S))^{\perp}$.

(ii) From the proposition 3.2.3, we have $(f^{-1}(T))^{\perp} \subseteq f^{-1}(T^{\perp})$, now we prove that $f^{-1}(T^{\perp}) \subseteq (f^{-1}(T))^{\perp}$. For any $x \in f^{-1}(T^{\perp})$, then $f(x) \in T^{\perp}$. For all $t \in f^{-1}(T)$, so $f(t) \in T$, $f(x \wedge t) = f(x) \wedge f(t) = 0$, since f is one-to-one, we have $x \wedge t = 0$, so $x \in (f^{-1}(T))^{\perp}$. Thus, $f^{-1}(T^{\perp}) \subseteq (f^{-1}(T))^{\perp}$. Hence, we have $(f^{-1}(T))^{\perp} = (f^{-1}(T^{\perp}))$.

4 Conclusion

Lattice Wajsberg algebra is a theoretical basis of non-classical algebra. As we know, in order to investigate the structure of an algebraic system, the ideals along with some properties will play an important role. In this paper, we have introduced the notion of annihilators, and proved that the annihilator is a *WI*- ideal in lattice Wajsberg algebra. Then we have given some properties of the annihilator and discussed the relationships between an annihilator and a *WI*-ideal, between the lattice implication homomorphism image of annihilator and an annihilator of lattice implication image in lattice Wajsberg algebras. Moreover, we have shown that the necessary and sufficient conditions of lattice implication homomorphism inverse image of annihilator. Finally, we discuss the lattice implication isomorphism inverse image of annihilator of lattice Wajsberg algebras.

Competing Interests

Authors have declared that no competing interests exist.

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