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Control Gain and Cost of Pinning Control for BA Scale-Free Networks

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Authors' contributions

This work was carried out in collaboration between both authors. Authors ZY and JC designed the study. Author ZY managed the literature searches, performed research and wrote the paper. All authors read and approved the final manuscript.

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ABSTRACT

This paper investigates the pinning control strategy of BA scale-free network through numerical analysis. A new cost function is defined to measure the cost of pinning control. Compared with the control gain and the cost of pinning nodes with smallest degrees or biggest degrees, an interesting result is that a smaller control gain and a lower cost are achieved by using the control scheme of pinning nodes with smallest degrees. Moreover, there is a minimal control cost by pinning nodes with smallest degrees and biggest degrees, respectively. The number of controllers by pinning nodes with smallest degrees is considered finally.

Keywords: BA scale-free networks; pinning control; control gain; control cost.

1. INTRODUCTION

There are many large-scale systems in nature and human societies which can be described by networks where nodes represent individuals in the system and edges represent the connection or interplay among the nodes [1-5]. More researchers crossing many fields of science, including physics, chemistry, biology, and mathematics are engaged in complex networks in recent years. There are more researches on controlling the dynamics of a network to a

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desired state such as an equilibrium state or a periodic orbit of the network.An efficient method for control, stabilization and synchronization of complex networks is pinning control [6-24], by which only a fraction of nodes or even a single node is controlled to steer the whole network. With feedback controller, the only problem is which nodes should be controlled. It has many studies focused on this problem [6-11] and the nodes with high degrees are usually selected to be controlled with larger control gain. This method has extended to directed networks or digraph [12-17] and it has been encouraged in recent years [18-21]. When considering the control cost [22-24], however, the nodes with smaller degrees should be pinned, in which the control cost is defined as sum of feedback gain times coupling strength of the network [22].

In this paper, the control strategy of pinning the nodes with smallest degrees and biggest degrees is analyzed with the eigenvalue of controlled network, respectively. From this analysis, for a fixed network, the nodes with smallest degrees should be chose to be controlled in the pinning control strategy. Furthermore, a new control cost function is defined to measure the cost of pinning control. Comparing with the control cost between pinning nodes with smallest degrees and biggest degrees, it shows pinning nodes with smallest degrees are more effective than pinning nodes with biggest degrees. Moreover, there is a minimal control cost under pinning nodes with smallest degrees and biggest degrees, respectively. That is, there is an optimal control gain when pinning “smallest” nodes or “biggest” nodes. Finally, we investigate the number of controllers by pinning nodes with smallest degrees by numerical simulation.

2. PINNING CONTROL

Suppose that an undirected and unweighted complex network consists of N identical linearly and diffusively coupled nodes with each node being an n-dimensional dynamical system. The state equations of this dynamical network are given by

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j, i = 1,2,\cdots,N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T \in R^n$ are the state variables of node i, $f(\cdot)$ is the dynamical function of an isolated node, c is the positive constant representing the coupling strength, $\Gamma = diag(r_1, r_2, \cdots, r_n)$ is the inner coupling matrix, if $r_i = 1$ means that two coupled nodes are linked through their i th state variables, otherwise $r_i = 0$.

The coupling matrix $A = (a_{ij}) \in R^{N\times N}$ represents the coupling configuration of the network. If there is a connection between node i and node j, then $a_{ij} = a_{ji} = 1$; Otherwise, $a_{ij} = a_{ji} = 0$; And the diagonal elements of matrix A are defined by

$$a_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ij}, i = 1,2,\cdots,N, \tag{2}$$

which ensures the diffusion that $\sum_{j=1}^{N} a_{ij} = 0$ for all rows. For connected networks, matrix A is semi-negative definite with zero eigenvalue of multiplicity one.

Suppose that we want to stabilize network (1) onto an equilibrium state defined by

$$x_1 = x_2 = \cdots = x_N = \bar{x}, f(\bar{x}) = 0. \tag{3}$$

To achieve the goal (3), we apply the pinning control strategy on a small fraction of the nodes in network (1). Without loss of generality, we rearrange the order of the nodes in the network, and let the first l nodes be controlled. Thus, the pinning controlled network can be described as

$$\begin{aligned} \dot{x}_i &= f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j + u_i, i = 1,2,\cdots,l, \\ \dot{x}_i &= f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j, i = l+1, l+2,\cdots,N, \end{aligned} \tag{4}$$

where

$$u_i = -cd_i\Gamma(x_i - \bar{x}), i = 1,2,\cdots,l, \tag{5}$$

are n -dimensional linear feedback controllers with all the control gains $d_i > 0$. For steering the controlled network (4) with (5) to its equilibrium state, we need the following lemma [7]:

Lemma: Consider the controlled network (4) with (5). Suppose that there exists a constant $\rho < 0$ such that $[Df(\bar{x}) + \rho\Gamma]$ is a Hurwitz matrix. If

$$c \geq \frac{|\rho|}{\min \lambda(-A+D)}, \tag{6}$$

then the equilibrium state $\bar{x}$ of the controlled network (4) is locally stable, where $D = diag(d_1, \cdots, d_l, 0, \cdots, 0) \in R^{N\times N}$ and $\lambda(-A+D)$ represents the eigenvalue of matrix $-A+D$.

Remark1: As shown in [7], if Γ is a positive definite matrix, then $|\rho| = L_f$ represents the globally stable about the equilibrium state $\bar{x}$ and $|\rho| = h_{\max}$ represents the locally stable, where $L_f > 0$ is the Lipchitz constant of $f(\cdot)$, $h_{\max} > 0$ is the maximum positive Lyapunov Exponent of chaotic system $\dot{x} = f(x)$.

Remark2: As shown in [8], a single controller can pin a coupled complex network to an equilibrium state if the coupling strength c is large enough.

3. PINNING STRATEGY

Two well-known strategy of pinning control schemes are randomly and specifically pinning. As discussed in [6-11], controllers are generally preferred to be added to nodes with larger degrees. However, it is also known that under such pinning schemes, the feedback gains d_i usually have to be relatively large. From the viewpoint of realistic applications, using large control gains is not expected and sometimes cannot be realized. Practically, a designed control strategy should be effective and also easily implementable. Inspired by [22] and [25,26], we introduce a new concept of cost function to evaluate the efficiency of our designed control scheme.

Definition (Control Cost): If under the controller (5), network (4) is stable. The Control Cost is defined as

$$CC = c\sum_{i=1}^{l} d_i \int_0^{+\infty} \| \Gamma(x_i - \bar{x}) \| \, dt, \tag{7}$$

where $\|\cdot\|$ represents the Euclid norm.

According to the Lemma, for a fixed network or a fixed coupling strength c, our aim is to select controlled nodes and make

$$\min \lambda(-A+D) \geq \frac{|\rho|}{c}, \tag{8}$$

for proper control gains d_i.

In the following simulations, we consider a 50-nodes BA scale-free network composed of Chen oscillators with

$$\Gamma = diag(1,1,1), |\rho| = h_{\max} = 2.0184, d_i = d.$$

There is only one node with biggest degree 22 and 19 nodes with smallest degree 3.

Figs. 1 and 2 show $\min \lambda(-A+D)$ versus the control gain when pinning the “biggest” node and all of the “smallest” nodes, respectively. We can see that for the same control gain d, $\min \lambda(-A+D)$ of controlling smallest degrees nodes is bigger than that of controlling the biggest degrees nodes. That is, for a fixed coupling strength, we should control the smallest degrees nodes with small control gain. Furthermore, we can see $\min \lambda(-A+D)$ has limitation for control gain $d \to \infty$. It is means that we not need relatively large control gain to control the whole network.

Figs. 3 and 4 show the process of controlling “biggest” node of degree 22 with $d = 30, c = 20$ and all “smallest” nodes of 3 with $d = 1, c = 20$, respectively. The control cost are 4.8079*10^6 and 1.5577*10^6, respectively. It can be seen controlling nodes with smaller degrees is more efficiency than controlling nodes with larger degrees even though we control 19 nodes with degree 3.

Remark 3: When controlling the “biggest” nodes with $c = 20$, the control gain $d = 7$ satisfies Eq. (8), which means we can control the whole networks for all $d \geq 7$. But for steering the network to its equilibrium state apparently in 10 time steps, we set $d = 30$ in Fig. 3.

4. CONTROL COST AND CONTROLLER NUMBER

The control cost versus control gain is simulated in Figs. 5 and 6 with controlling the “biggest” node and all “smallest” nodes, respectively. According to the two figures, it shows that the control cost decreases rapidly at first, then reaches a minimal value and increases slowly with the increase of control gains at last. The critical control gain corresponding to the minimal control cost can be chosen as the optimal control gain in the sense of consumed energy. The optimal control gain and minimal control cost are 240 and 1.5385*10^6 in Fig. 5, 11 and 0.6414*10^6 in Fig. 6, respectively.

As shown above, the control scheme of pinning nodes with smallest degrees can be much more effective than that of pinning nodes with biggest degrees. However, because the number of nodes with smallest degrees in a network is often large, the number of controllers to be applied should also be large if all these nodes are controlled. Thus, when pinning a complex networks with fixed coupling strength and control gain, research on the number of controllers by pinning smallest nodes is interesting and valuable.

Fig. 7 shows $\min\lambda(-A+D)$ versus the number of controllers by pinning nodes with smallest degrees when $d=5, c=20$. We can see that $\min\lambda(-A+D)$ increases rapidly with the increase of controller number. It is shown when pinning 5 “smallest” nodes, Eq. (8) is satisfied, which means we can steer the whole network to its equilibrium state by only pinning 5 nodes with smallest degrees. In Fig. 8, we show the process of controlling 5 “smallest” nodes of degree 3 based on controlled network system (4) of Chen oscillators with controller (5) when $d=5, c=20$. It is shown that the network is quickly steered to its equilibrium state just in 1 time step.

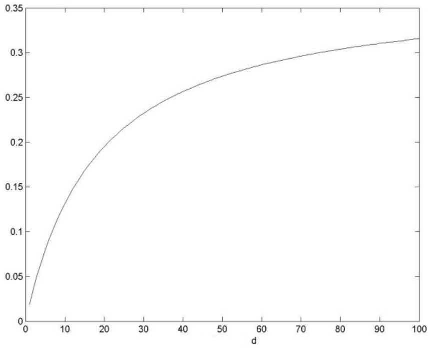

Fig. 1. $\min\lambda(-A+D)$ **versus the control gains when pinning the “biggest” node**

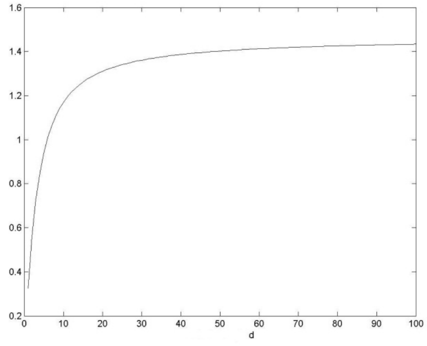

Fig. 2. $\min\lambda(-A+D)$ **versus the control gains when pinning the “smallest” nodes**

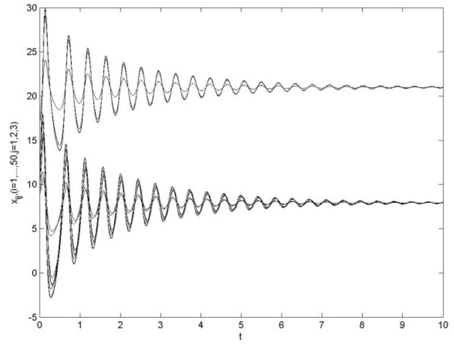

Fig. 3. Convergence of the network when controlling “biggest” with

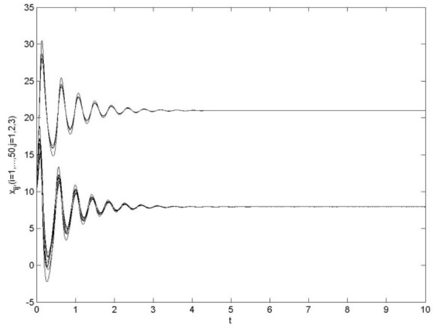

Fig. 4. Convergence of the network when controlling “smallest” nodes with

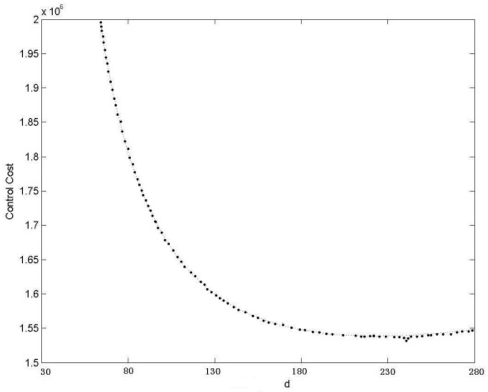

Fig. 5. Control cost of pinning the "biggest" nodes

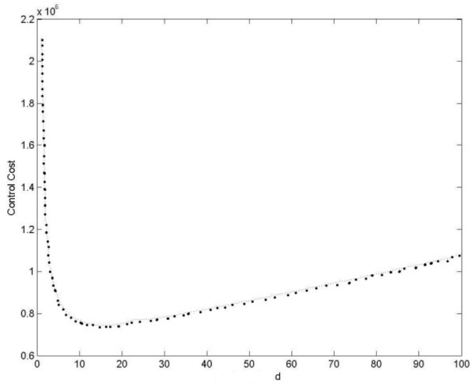

Fig. 6. Control cost of pinning the "smallest" nodes

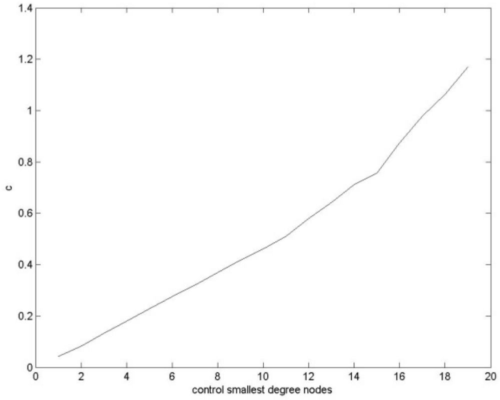

Fig. 7. $\min \lambda(-A+D)$ **versus the number of controllers**

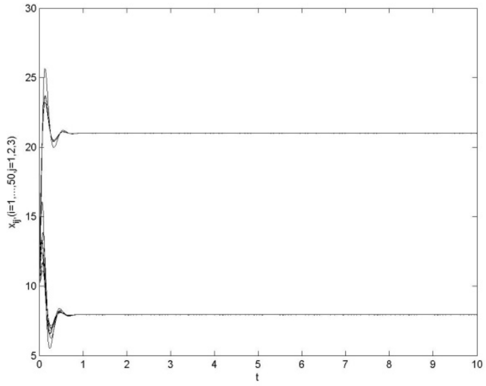

Fig. 8. Convergence of pinning 5 "smallest" nodes

5. CONCLUSION

In this paper, pinning control for BA scale-free networks has been further investigated. According to the eigenvalue analysis, an interesting result is that controlling the nodes with smallest degrees is more effective than controlling the biggest ones. Furthermore, we define a new control cost function to evaluate the effectiveness of each control scheme. According to the simulation of control cost function, we also see pinning nodes with smallest degrees is more effective than pinning biggest ones. Moreover, the control cost function has minimum value not only by pinning nodes with smallest degrees but also by pinning nodes with biggest degrees. In the end, the number of controllers by pinning nodes with smallest degrees is investigated, which shows we should not pin every node with smallest degrees in practice.

In the future study, we attempt to give theoretical analysis of our results, especially about the relationship between control cost and controller number. Moreover, we will investigate the control cost of pinning control on directed networks.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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