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Effects of Radiation and Soret in the Presence of Heat Source/Sink on Unsteady MHD Flow Past a Semi-infinite Vertical Plate

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Abstract

The paper deals with the effects of radiation and Soret number variation in the presence of heat source/sink on unsteady laminar boundary layer flow of a chemically reacting fluid along a semi-infinite vertical plate, taking the term Eckert number into account. A magnetic field of uniform strength is applied normal to the flow. The governing boundary layer equations are solved numerically, using Crank-Nicholson method and the simulation is carried out by coding in C-Programme. Graphical results for velocity, temperature and concentration fields and tabular values of Skin-friction, Nusselt and Sherwood numbers are presented and discussed at various parametric conditions. From this study, it is found that the Skin–friction, Nusselt number, temperature and velocity of the fluid increase in the presence heat source and for increasing values Eckert number (Ec).

Keywords: Thermal diffusion (Soret), magnetic field, chemical reaction, Crank-Nicholson method, chemical reaction, radiative heat flux, heat source/sink.

1 Introduction

Several authors have dealt with heat flow and mass transfer over a vertical porous plate with variable suction, heat absorption/ generation, radiation and chemical reaction. Actually, many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment, Nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases one has to take into account the effects of radiation.

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NOMENCLATURES

ρ	*Density*
C_p	*Specific heat at constant pressure*
ν	*Kinematic viscosity*
k	*Thermal conductivity*
Gr	*Free convection parameter due to temperature*
Gm	*Free convection parameter due to concentration*
A	*Suction parameter*
n	*A constant exponential index*
D	*Molar diffusivity*
NR	*Thermal radiation parameter*
M	*Magnetic parameter*
σ	*Electrical conductivity*
D_m	*Mass diffusion coefficient*
K_T	*Thermal diffusion ratio*
T_m	*Mean fluid temperature*
S	*Heat source/sink parameter*
Ec	*Eckert number*
β^*	*Volumetric coefficient of expansion with concentration*
β	*Coefficient of volumetric thermal expansion of the fluid*
Kr	*Chemical reaction rate constant*
Sc	*Schmidt number*
T	*Temperature*
Pr	*Prandtl number*
$\in$	*Small reference parameter << 1*
So	*Soret number*
U_o	*Mean velocity*
a_R	*Rosseland radiation absorbtivity*

Several authors have considered effects of radiation on Newtonian flows. Perdikis et al. [1] illustrated the heat transfer of a micropolar fluid in the presence of radiation. Raptis [2] studied the effect of radiation on the flow of a micro-polar fluid past a continuous moving plate. Raptis et al. [3] studied the viscoelastic flow by the presence of radiation. Elbashbeshby [4] and Kim et al. [5] have reported the effects of radiation on the mixed convection flow of a micro-polar fluid. Chamkha et al. [6] analyzed the effects of radiation on free convection flow past a semi infinite vertical plate with mass transfer. Ganesan and Loganathan [7] studied the radiation and Mass transfer effects on flow of a viscous incompressible fluid past a moving cylinder. Ramachandra Prasad et al. [8] considered the effects of radiation and Mass transfer on two dimensional flow past an infinite vertical plate. Raptis [9] discussed the effect of radiation on steady flow of a viscous fluid through a porous medium bounded by a porous plate subjected to a constant suction velocity.

Abdus Sattar and Hamid Kalim [10] investigated the unsteady free convection interaction with thermal radiation in the boundary layer flow past a vertical porous plate. Makinde [11] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid. Prakash and Ogulu [12] have studied the effects of thermal radiation, time-dependent suction and chemical reaction on two-dimensional flow of an incompressible Boussinesq fluid. Moreover, when the radiative heat transfer takes place, the fluid involved can be electrically conducting in the sence that it is ionized owing to high operating temperature. In such case one cannot neglect the effect of magnetic field on the flow field. Taking magnetic field into account, Sharma et al. [13] discussed the effect of radiation on free convective flow, along a uniform moving porous vertical plate. Sharma et al. [14] have reported on the radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface with ohmic heating. Chaudhary and Preethi Jain [15] presented an analysis to study the effects of radiation on the hydromagnetic free convection flow of an electrically conducting micropolar fluid past a vertical porous plate through a porous medium in slip-flow regime. Takhar et al. [16] considered the effect of radiation on free-convection flow of a radiation gas past a semi infinite vertical plate in the presence of magnetic field. Raptis and Massalas [17] studied the magneto-hydrodynamic flow past a plate by the presence of radiation. Sudheer Babu and Satyanarayana [18] discussed the effects of the chemical reaction and radiation absorption in the presence of magnetic field on free convection flow through porous medium with variable suction. Dulal Pal et al. [19] has made the Perturbation analysis to study the effects thermal radiation and chemical reaction on magneto-hydrodynamic unsteady heat and mass transfer in a boundary layer flow past a vertical permeable plate in the slip flow regime. Ibrahim et al. [20] analysed the effects of the chemical reaction and radiation absorption on transient hydro-magnetic free-convention flow past a semi infinite vertical permeable moving plate with wall transpiration and heat source.

Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight, several investigators like Eckert and Drake [21], Dursunkaya and Worek [22], Anghel et al. [23], Olanrewaju and Makinde [24], Makinde [25], have studied and reported the results for these flows. In addition to this, Anand Rao et al. [26] analysed the effects of Viscous dissipation and Soret on an unsteady two-dimensional laminar mixed convective boundary layer flow of a chemically reacting viscous incompressible fluid, along a semi-infinite vertical permeable moving plate. Recently, The Soret and Dufour effects on unsteady MHD mixed convection flow past an infinite radiative vertical porous plate embedded in a porous medium in the presence of chemical reaction have been studied Sharma et al. [27]. More recently, Srihari and kesavareddy [28] have made the investigation to study the effects of Soret and Magnetic field on unsteady laminar boundary layer flow of a radiating and chemically reacting viscous incompressible fluid along a semi-infinite vertical plate.

In most of the earlier studies there seems to be no significant consideration of the effects radiation and Soret in the presence of heat source/sink, which plays a vital role in maintaining heat transfer at desired level in the applications of Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles. Fluid supports an exothermic chemical or nuclear reaction is very common today and the correct processes design requires accurate correlation for the heat transfer coefficients at the boundary surfaces. Despite of its increasing importance in technological and physical problems, the magneto-hydrodynamic flow of a dissipative fluid past an infinite plate have received much attention because of the non-linearity of the governing equations.

Hence based on the above discussion in the present paper a numerical attempt is made to study the effects of radiation and Soret number variation in the presence of heat source/sink on unsteady laminar boundary layer flow of chemically reacting incompressible viscous fluid along a semi-infinite vertical plate, taking the term viscous dissipation (Eckert number) in to account. A magnetic field of uniform strength is applied normal to the fluid flow.

2 Mathematical Formulation

An unsteady laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate, in the presence of thermal and concentration buoyancy effects has been considered. The x'- axis taken along the plate in the vertically upward direction and y'-axis normal to it. A magnetic field of uniform strength applied along y' -axis. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and *t'* only. A time dependent suction velocity is assumed normal to the plate. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are:

Continuity

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u' \tag{2}$$

Energy

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q(T - T_\infty)}{\rho c_p} \tag{3}$$

Mass transfer

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y} = \upsilon\frac{\partial^2 C}{\partial y'^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y'^2} - K_r'(C - C_\infty) \tag{4}$$

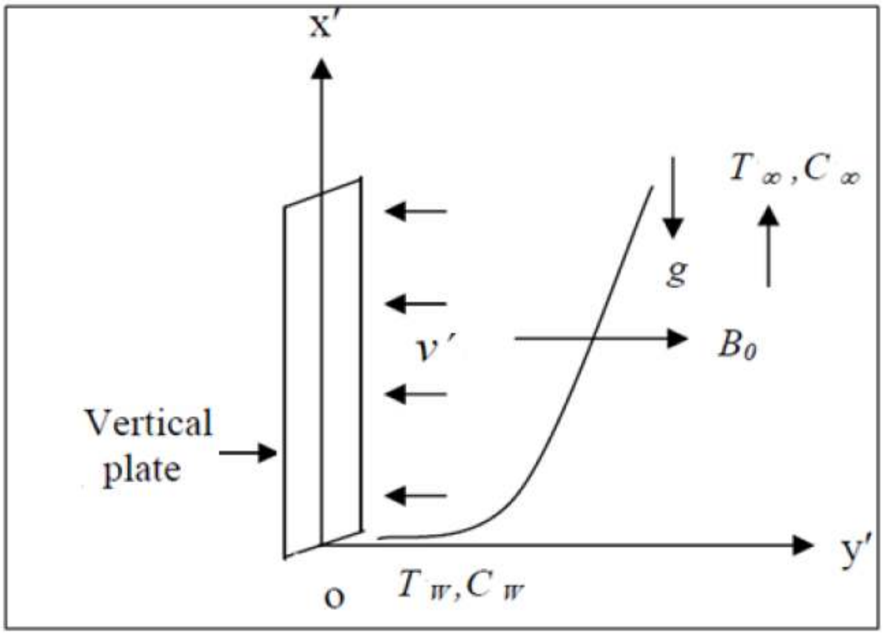

Diagram 2.1. Schematic diagram of flow geometry

The radiative flux (q_r) by using the Rosseland approximation [29] is given by

$$q_r = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y'} \tag{5}$$

The boundary conditions suggested by the physics of the problem are

$$u' = U_0,\ T = T_W + \varepsilon(T_W - T_\infty)e^{n't'},\ C = C_W + \varepsilon(C_W - C_\infty)e^{n't'} \quad at\ \ y' = 0$$

$$u' \to 0, \quad T \to T_\infty, \quad C \to C_\infty \qquad as\ \ y' \to \infty \tag{6}$$

It has been assumed that the temperature differences within the flow are sufficiently small and T^4 may be expressed as a linear function of the temperature T. This is accomplished by expanding T^4 in a Taylor series about $\boldsymbol{T_\infty}$ as follows [30]

$$f(T) = f(T_\infty) + (T - T_\infty)f'(T_\infty) + \frac{(T - T_\infty)^2}{2!}f''(T_\infty) + \ldots, \tag{7}$$

Where $f(T)=T^4$ then $f'(T)=4T^3$, $f''(T)=12T^2$
Simplifying (7), we get

$$T^4 = T_\infty^4 + 4(T - T_\infty)T_\infty^3 + 12\frac{(T - T_\infty)^2}{2!}T_\infty^2 + \ldots$$

In the above **Taylors** expansion, neglecting the higher order terms, we have

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using (8) in (5) and then (5) in (3), equation of Energy (3) is transformed to

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q(T - T_\infty)}{\rho c_p} \tag{9}$$

Integration of continuity equation (1) for variable suction velocity, normal to the plate gives

$$v' = -U_0\left(1 + \varepsilon A e^{n't'}\right) \tag{10}$$

where A is the suction parameter and εA is less than unity. U_o is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is towards the plate.

Using the equation (10) and introducing the following non-dimensional quantities

$$u = \frac{u'}{U_0}, \quad t = \frac{U_0^2 t'}{\upsilon}, \quad n = \frac{\upsilon n'}{U_0^2}, \quad y = \frac{y' U_0}{\upsilon}$$

$$Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{U_0^3}, \quad Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{U_0^3}, \quad S = \frac{Q\upsilon}{\rho C_p U_0^2}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad So = \frac{D_m k_T (T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)}, \quad Ec = \frac{U_0^2}{C_p (T_w - T_\infty)}$$

$$Kr = \frac{K_r' \upsilon}{U_0^2}, \quad NR = \frac{16\sigma^* T_\infty^3}{3a_R k}, \quad \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\upsilon}{D} \tag{11}$$

into equations (1),(2),(4) and (9), we get the equations in non-dimensional form as follows

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - Mu \tag{12}$$

$$\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\theta}{\partial y}=\left(\frac{1+NR}{Pr}\right)\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2+S\theta \tag{13}$$

$$\frac{\partial\phi}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}+So\frac{\partial^2\theta}{\partial y^2}-Kr\,\phi \tag{14}$$

with the boundary conditions

$$\begin{aligned} &u=1,\quad \theta=1+\varepsilon e^{nt},\quad \phi=1+\varepsilon e^{nt} && \text{at } y=0\\ &u\to 0,\quad \theta\to 0,\quad \phi\to 0 && \text{as } y\to\infty \end{aligned} \tag{15}$$

In order to establish a mathematical convenience of converging the solution at a finite point **(**$\eta\to 1$**)**, equations (12)-(15) should be transformed to a new system of coordinates. So, employing the transformation $\eta=1-e^{-y}$ on the equations (12)-(15), the following are obtained

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial u}{\partial \eta}=\left((1-\eta)^2\frac{\partial^2 u}{\partial\eta^2}-(1-\eta)\frac{\partial u}{\partial\eta}\right)+Gr\theta+Gm\phi-Mu \tag{16}$$

$$\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial\theta}{\partial \eta}=\frac{1+NR}{Pr}\left((1-\eta)^2\frac{\partial^2\theta}{\partial\eta^2}-(1-\eta)\frac{\partial\theta}{\partial\eta}\right)+Ec\left((1-\eta)\frac{\partial u}{\partial\eta}\right)^2+S\theta \tag{17}$$

$$\frac{\partial\phi}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial\phi}{\partial \eta}=\frac{1}{Sc}\left((1-\eta)^2\frac{\partial^2\phi}{\partial\eta^2}-(1-\eta)\frac{\partial\phi}{\partial\eta}\right)+So\left((1-\eta)^2\frac{\partial^2\theta}{\partial\eta^2}-(1-\eta)\frac{\partial\theta}{\partial\eta}\right)-Kr\,\phi \tag{18}$$

with corresponding boundary conditions

$$\begin{aligned} &u=1:\quad \theta=1+\varepsilon e^{nt},\quad \phi=1+\varepsilon e^{nt} && at\quad \eta=0\\ &u\to 0:\ \theta\to 0,\qquad \theta\to 1 && as\quad \eta\to 1 \end{aligned} \tag{19}$$

3 Method of Solution

The equations (16)-(18) are coupled, non-linear partial differential equations whose exact solution is difficult to obtain, hence the problem is solved numerically, using the following finite difference formulae

$$\frac{\partial f}{\partial t}=\frac{f_i^{j+1}-f_i^{j}}{\Delta t}\quad,\quad \frac{\partial f}{\partial\eta}=\frac{f_{i+1}^{j}-f_i^{j}}{\Delta\eta}$$

$$\frac{\partial^2 f}{\partial \eta^2} = \frac{1}{2}\left(\frac{f_{i-1}^{j} - 2 f_i^{\,j} + f_{i+1}^{j}}{(\Delta\eta)^2} + \frac{f_{i-1}^{j+1} - 2 f_i^{\,j+1} + f_{i+1}^{j+1}}{(\Delta\eta)^2} \right),$$

(where f stands u,θ and Φ)

into the equations (16), (17) and (18) and simplifying according to the **Crank and Nicholson method** , we get the following system of equations

$$-P_3\, r\, u_{i-1}^{j+1} + (1 + 2P_3\, r)u_i^{\,j+1} - P_3\, r\, u_{i+1}^{j+1} = E_i^{\,j} \tag{20}$$

$$-P_3 P_4\, r\, \theta_{i-1}^{j+1} + (1 + 2P_3 P_4\, r)\theta_i^{\,j+1} - P_3 P_4\, r\, \theta_{i+1}^{j+1} = F_i^{\,j} \tag{21}$$

$$-\frac{P_3 r}{Sc}\phi_{i-1}^{j+1} + \left(1 + \frac{2P_3 r}{Sc}\right)\phi_i^{\,j+1} - \frac{P_3 r}{Sc}\phi_{i+1}^{j+1} = H_i^{\,j} \tag{22}$$

with boundary conditions in finite difference form

$$\begin{aligned} &u\,(0, j) = 1, \quad \theta(0, j) = 1 + \varepsilon\exp(n.\, j.\Delta t), \quad \phi = 1 + \varepsilon\exp(n.\, j.\,\Delta t), \quad \forall\, j \\ &u(10, j) \to 0, \quad \theta(10, j) \to 0, \quad \phi(10, j) \to 1 \quad \forall\, j \end{aligned} \tag{23}$$

where

$$E_i^j = P_3 r\, u_{i-1}^j - (1 - P_1P_2\, r\Delta\eta - 2P_3 r + P_2 r\Delta\eta - M\,\Delta t)u_i^j + (P_1\, P_2 r\Delta\eta + P_3 r - P_2 r\Delta\eta)u_{i+1}^j + Gr\Delta t\theta_i^j + Gm\Delta\, t\phi_i^j$$

$$F_i^j = P_3P_4\, r\theta_{i-1}^j + (1 - P_1P_2\, r\Delta\eta - 2P_3P_4\, r + P_2P_4\, r\Delta\eta + S\,\Delta t\,)\theta_i^j + (P_1P_2\, r\Delta\eta + P_3P_4\, r - P_2P_4\, r\Delta\eta)\theta_{i+1}^j + 2P_3 Ec\left(\frac{u_{i+1}^j - u_i^j}{\Delta\eta}\right)^2$$

$$\begin{aligned} H_i^j = \frac{P_3 r}{Sc}\,\phi_{i-1}^j + \left(1 + P_1P_2\, r\Delta\eta - \frac{2P_3 r}{Sc} + \frac{P_2 r\Delta\eta}{Sc} - Kr\Delta t\right)\phi_i^j + \left(\frac{P_3\, r}{Sc} - P_1\, P_2 r\Delta\eta - \frac{P_2\, r\Delta\eta}{Sc}\right)\phi_{i+1}^j \\ + (2P_3\, rS_O - S_O\, P_1\, r\Delta\eta)\theta_{i+1}^j + (S_O\, P_1\, r\Delta\eta - 4P_3\, r\, S_O)\,\theta_i^j + 2P_3\, r\, S_O\, \theta_{i-1}^j \end{aligned}$$

$$P_1 = 1 + \in Ae^{\,n\,j\,\Delta t},\; P_2 = 1 - i\,\Delta\eta,\; P_3 = \frac{(1 - i\,\Delta\eta)^2}{2},\; P_4 = \frac{1 + NR}{\Pr}$$
$$r = \Delta t/(\Delta\eta)^2$$

Here $\Delta\eta$ and Δt are mesh sizes along η and time *t*- direction, respectively. Index *i* refers to space and *j* for time.

To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to **η** and ***t***-axe with $\Delta\eta=0.1$ and $\Delta t=0.005$. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The finite-difference equations at every internal nodal point on a particular *n*-level constitute a tri-diagonal system of equations. These equations are solved by using the Thomas algorithm [31] and the simulation is carried out by coding in C-Program. In order to prove the convergence of finite difference scheme, the computation is carried out for slightly changed values of Δη and Δt, and the iterations on until a tolerance of 10^{-8} is attained. Negligible change is observed in the values of u, θ and ϕ.Thus, it is concluded that, the finite difference scheme is convergent and stable.

From the technological point of view, after knowing the velocity, temperature and concentration profiles, it is important to know the skin-friction, rate of heat and mass transfer between the plate and the fluid.

3.1 Skin-friction

The Skin friction coefficient τ is given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0}, \tag{24}$$

3.2 Nusselt Number

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} \tag{25}$$

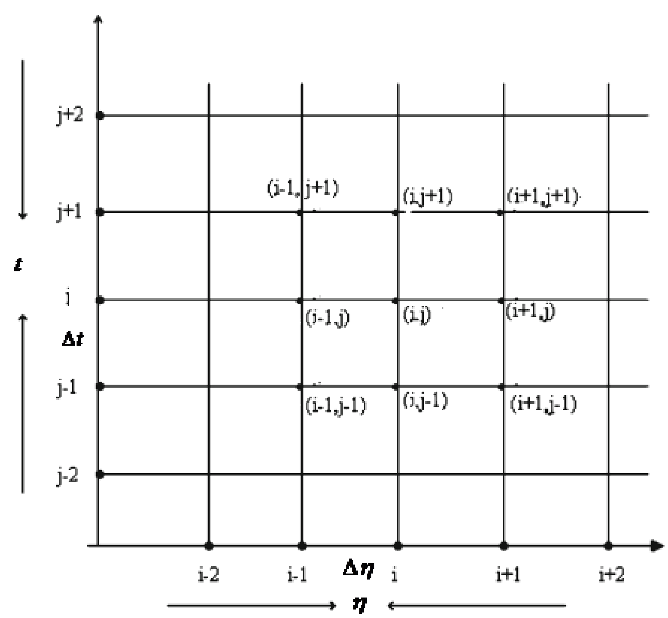

Diagram. 3.1. Grid meshing for finite difference method

3.3 Sherwood Number

The rate of mass transfer in terms of modified Nusselt number is given by

$$Sh = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \phi}{\partial \eta}\right|_{\eta=0} \tag{26}$$

4 Results and Discussion

In order to get a physical insight into the problem, the numerical calculations for the distribution of the velocity, temperature, concentration, skin-friction coefficient, rate of heat and mass transfer across the boundary layer for various values of flow parameters such as heat source and sink parameter (*S*), Grashof number (*Gr*), Modified Grashof number (*Gm*), Magnetic parameter (*M*), Prandtl number (*Pr*) Schmidt number (*Sc*), Radiation parameter (*NR*), Soret number (*So*), Eckert number (*Ec*) and chemical reaction parameter (*Kr)* have been carried out. During the course of numerical calculations, to be realistic, the values of Prandtl number (*Pr*) are chosen to be 0.71, 7.0 and 11.4 representing air, water at 20^0 C and water at 4^0 C respectively. Also *Pr* =1.0 is chosen corresponding to electrolytic solution as the propagation of thermal energy through electrolytic solution in the presence of heat source, sink and magnetic field has wide range of applications in chemical engineering, aeronautical engineering and atomic propulsion science.

The effects of *Gr* and *Gm* in the presence of heat source on velocity field u are shown in the (Figs. 1 and 2) respectively. It is observed that an increase in *Gr* and *Gm* leads to increase in the velocity of the flow because favourable buoyancy force accelerates the flow. It is also observed that as the values of *Gr* (or) *Gm* increases, the peak value of the velocity increases rapidly near the wall of the plate and then decay to the free stream velocity. Further, it is interesting to note that the fluid velocity increases in the presence of heat source, compared to absence of heat source.
Effects of *M* and *So* in the presence of heat source on velocity field u are shown in (Fig. 3) and (Fig. 4), respectively. It can be inferred from figures that an increase in *So* leads to increase in the velocity, but an increase in *M* leads to decrease in the velocity. The presence of magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied normal to the fluid flow. This type of resistive force tends to slow down the flow field.

(Fig. 5) depicts the velocity profile for various values of heat source and sink parameter (*S*) while (Fig. 10) shows the temperature profile for different values of Eckert number (Ec) and heat source/sink parameter (*S*). It is evident from the figures that the temperature and velocity increase with an increase in the heat source parameter (*S*). This result qualitatively agrees with expectation since the effect of heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid and also increasing its velocity. It is also noted that temperature and velocity of the fluid decrease in the presence of heat sink as heat absorption is to decrease the rate of heat transfer to the fluid. The analysis of (Fig. 10) reveals that the effect of increasing values of Eckert number is to increase temperature distribution in the flow region. This is due to the fact that heat energy is stored in the fluid due to the frictional heating.

(Figs 6 and 9) are drawn for various values of *Pr* on velocity and temperature field respectively. A comparative study of the graph reveals that the velocity and temperature of the fluid decrease as the value of Prandtl number increases. This is a good agreement with physical fact that an increase in *Pr* leads to decrease in the thermal boundary layer thickness. The reason underlying such behavior is that the higher Prandtl number fluid has relatively lower thermal conductivity. This results in the reduction of the thermal boundary layer thickness and thereby decreasing its velocity. From (Fig. 6) it is cleared that velocity of the fluid increases in the presence of source parameter while in the presence of sink it decreases.

From (Figs. 7 and 8), an important observation noted that the temperature and velocity increases as the radiation parameter increases. This result can be explained by the fact that an increase in the radiation parameter $NR = 16\sigma^{*}T_{\infty}^{3} / 3k\,a_{R}$, forgiven k and T_{∞}, means a decrease in the Roseland radiation absorbtivity (a_R). In view of equations (3) and (5), it is concluded that the divergence of the radiation heat flux $\partial q_r/\partial y^*$, increases as ($a_R$) decreases and this means that the rate of radiative heat, transferred to the fluid increases and consequently the fluid temperature and hence the velocity of its particles also increases.

(Figs. 11 and 12) display the effects of *So* and *Kr* on concentration field respectively. A comparison of the curves in the figures shows that a decrease in the concentration distribution with the increase of *Kr*. From the graph, it is found that an increase in the Soret number So results in an increase in the concentrations of the fluid while an increasing values of the chemical reaction parameter there is a fall in the concentration of the fluid.

Skin-friction coefficient, Nusselt and Sherwood numbers are presented (Tables 1, 2 and 3) respectively, for the both the cases of presence/absence of heat source and Eckert number. A comparative study of the numerical results in (Tables 1 and 2), reveal that Skin–friction and Nusselt number increase in the presence of heat source and Eckert number. This due to the fact that internal heat generation is to increase the rate of heat transfer to the fluid and increasing values Eckert number is to increase the temperature due to the frictional heating. Further, it is interesting to note that Skin-friction increases with increasing values of *So*, *NR*, *Gr* and *Gm*, but it decreases with increasing values of *M*, *Pr* and *Sc*. From table (3), it is observed that Sherwood number decreases in the presence of heat source and Eckert number.

Table 1. Effects of Gr, Gm, Pr, Sc, Kr, NR, So and M on skin-friction coefficient

Gr	**Gm**	**Pr**	**Sc**	**Kr**	***NR***	***So***	**M**	τ **S=0.0,Ec=0.0 Previous [28]**	τ **S=2.0,Ec=0.5 Present**
5.0	5.0	0.71	0.24	0.5	0.5	0.0	0.0	1.202	1.4032
5.0	5.0	0.71	0.24	0.5	0.5	0.0	2.0	0.557	0.7413
5.0	5.0	0.71	0.24	0.5	0.5	2.0	2.0	0.8394	0.9721
5.0	5.0	0.71	0.24	0.5	1.0	2.0	2.0	0.9183	1.0523
5.0	5.0	0.71	0.6	0.5	0.5	2.0	2.0	0.7601	0.8423
5.0	5.0	7.0	0.24	0.5	0.5	2.0	2.0	0.3156	0.3838
5.0	10.0	0.71	0.24	0.5	0.5	2.0	2.0	2.6542	2.7352
10.0	5.0	0.71	0.24	0.5	0.5	2.0	2.0	2.0447	2.3597

Table 2. Effects of NR and Pr on Nusselt-number

NR	*Pr*	*Nu* S=0.0, Ec=0.0 Previous [28]	*Nu* S=2.0, Ec=0.5 Present
0.0	0.71	-1.4771	-1.0807
0.5	0.71	- 1.1621	-0.8230
0.5	7.0	- 4.2655	-3.6770
0.5	11.4	-5.3251	-4.7594

Table 3. Effects of Sc, Kr and So on Sherwood number

Sc	Kr	So	Sh S=0.0, Ec=0.0 Previous[28]	Sh S=2.0, Ec=0.5 Present
0.24	0.5	0.0	-0.5931	-0.59393
0.24	0.5	2.0	-0.1156	-0.37159
0.24	1.0	2.0	-0.1858	-0.43987
0.6	0.5	2.0	-0.00291	-0.55924

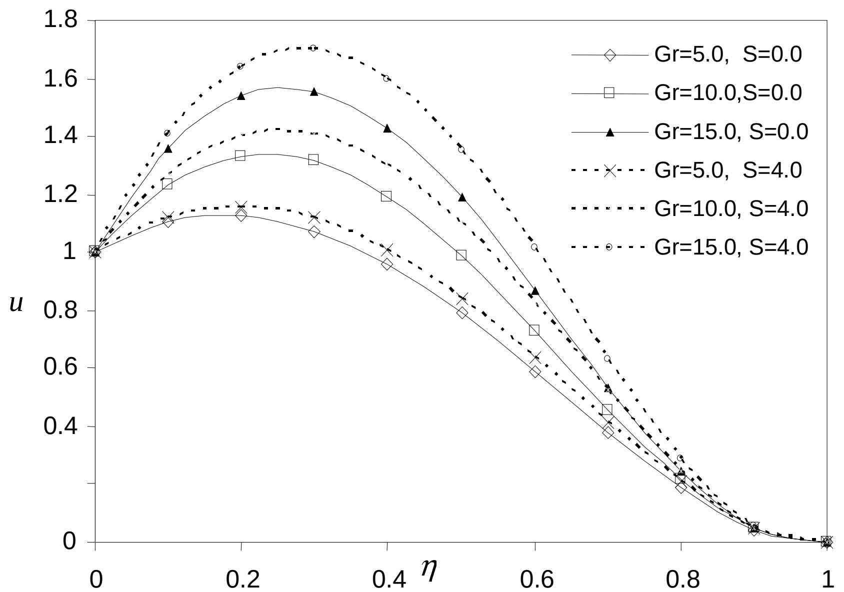

Fig 1. Effect of Grashof number Gr on velocity field u in the presence/absence of heat source
(Gm=5.0, NR=0.5, Pr=0.71, Sc=0.22, Kr=0.5.So=1.0, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

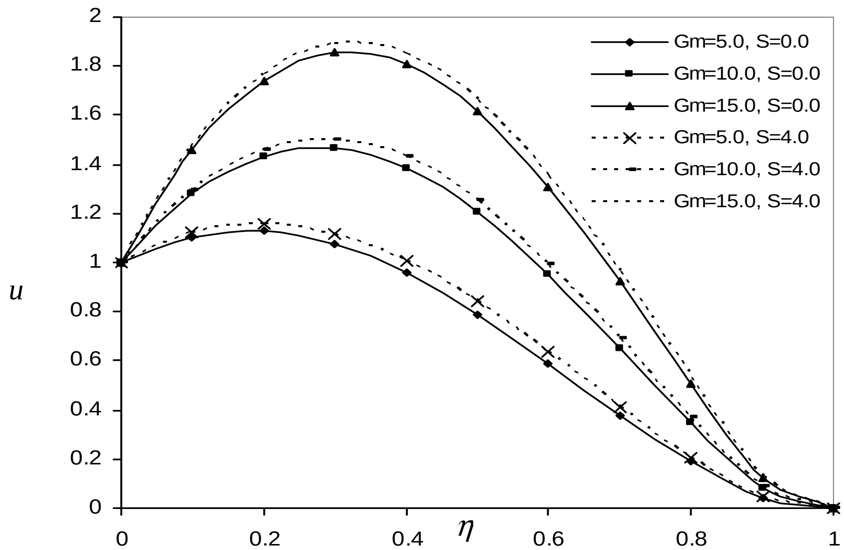

Fig. 2. Effect of Modified Grashof number Gr on velocity field u in the presence/absence of heat source

(Gr=5.0, NR=0.5, Pr=0.71, Sc=0.22, Kr=0.5.So=1.0, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

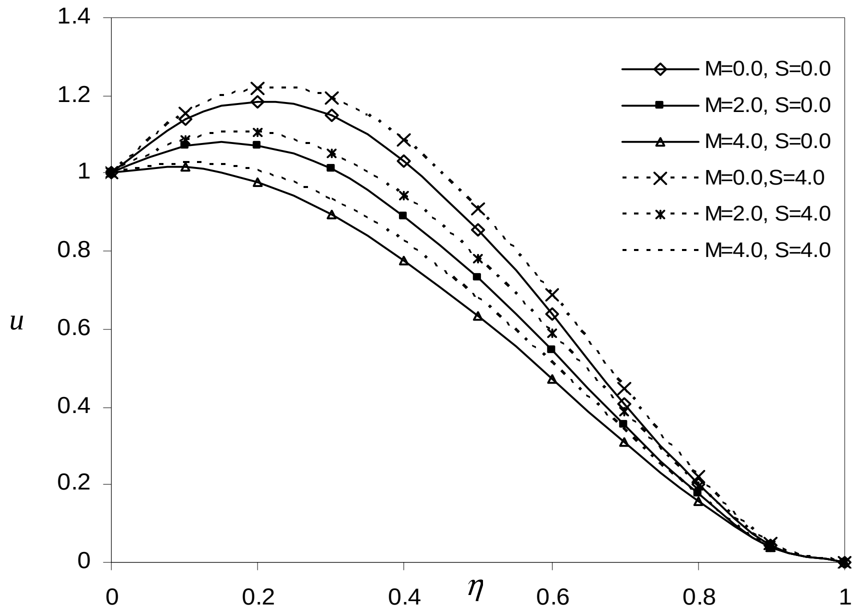

Fig. 3. Effect of Magnetic parameter M on velocity field u in the presence/absence of heat source

(Gr=5.0,Gm=5.0,NR=0.5,Pr=0.71,Sc=0.22,Kr=0.5.Ec=0.5,So=1.0,ε=0.01 n=0.1, A=0.3 and t=1.0)

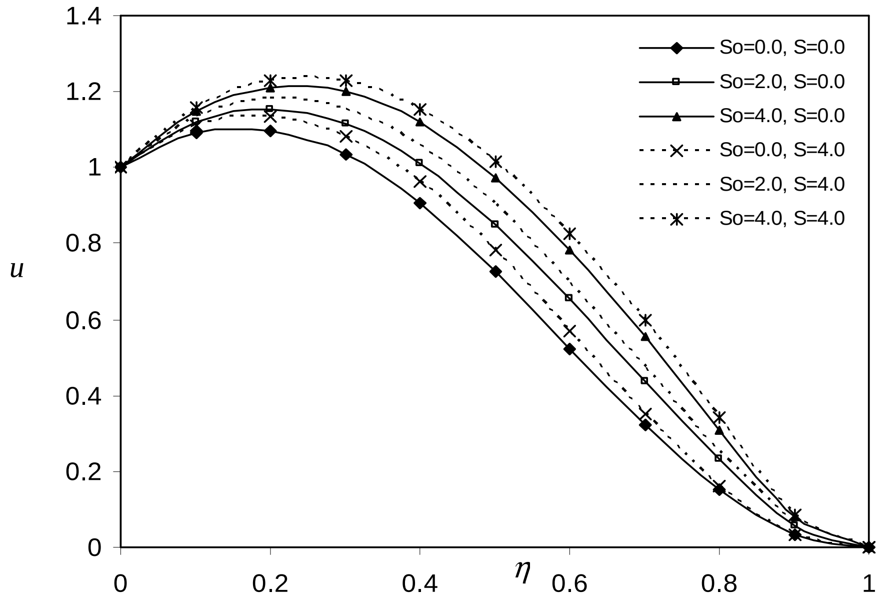

Fig. 4. Effect of Soret number So on velocity field u in the presence/absence of heat source
(Gr=5.0,Gm=5.0,NR=0.5,Pr=0.71,Sc=0.22,Kr=0.5.Ec=0.5,M=1.0,ε=0.01, n=0.1, A=0.3 and t=1.0)

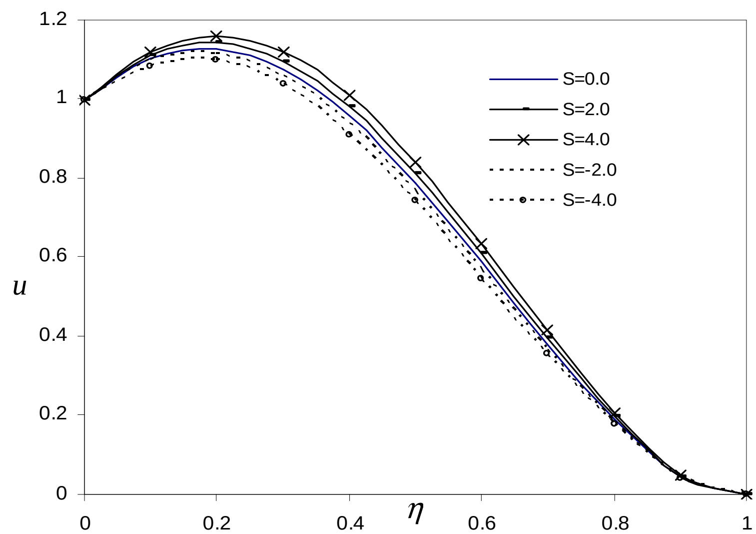

Fig. 5. Effect of heat source/sink on velocity field u
(Gr=5.0,Gm=5.0,NR=0.5,Pr=0.71,Sc=0.22,Kr=0.5.So=1.0,Ec=0.5,M=1.0,ε=0.01, n=0.1, A=0.3 and t=1.0)

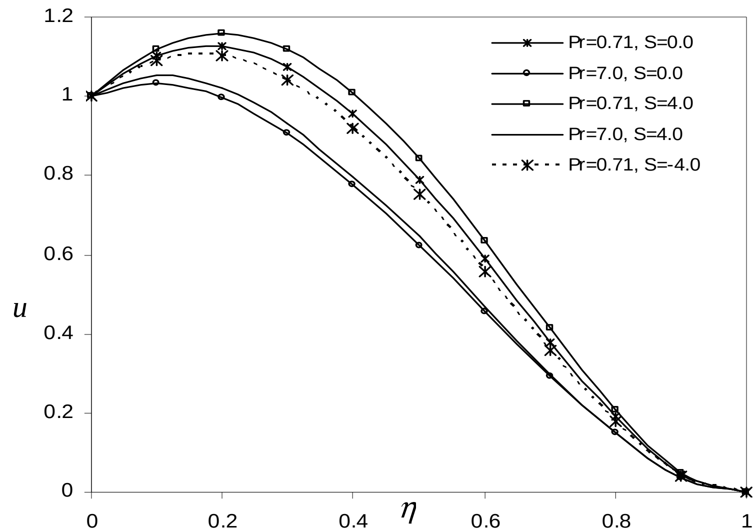

Fig. 6. Effect of Prandtl number Pr on velocity field u in the presence/absence of heat source/sink

(Gr=5.0,Gm=5.0,NR=0.5,So=1.0,Sc=0.22,Kr=0.5.Ec=0.5,M=1.0,ε=0.01, n=0.1, A=0.3 and t=1.0)

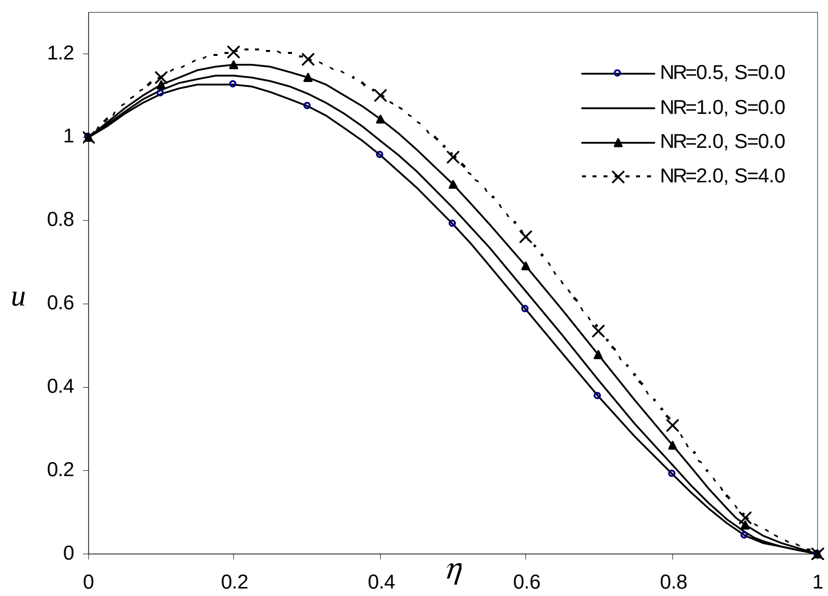

Fig. 7. Effect of Radiation NR on velocity field u in the presence/absence of heat source/sink

(Gr=5.0, Gm=5.0, So=1.0, Pr=0.71, Sc=0.22, Kr=0.5.Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

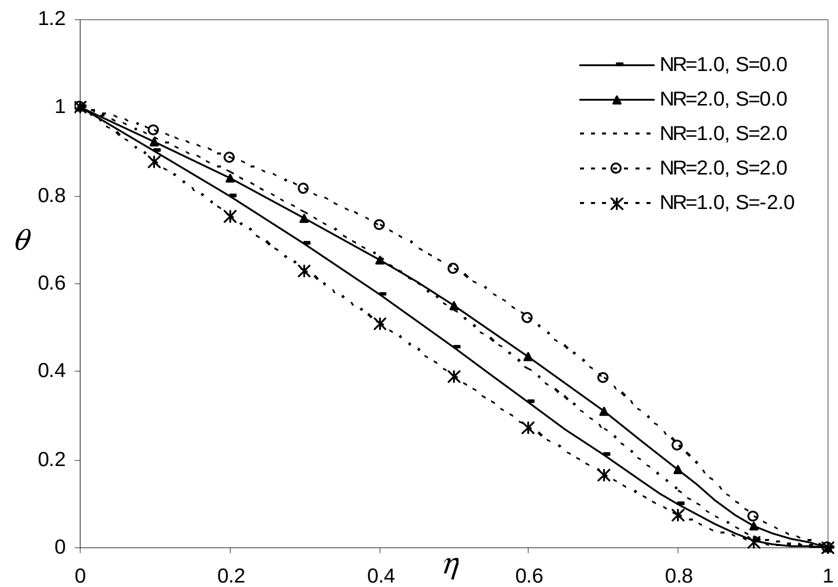

Fig. 8. Effect of Radiation NR on temperature field in the presence/absence of heat source/sink
(Gr=5.0, Gm=5.0, So=1.0, Pr=0.71, Sc=0.22, Kr=0.5, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

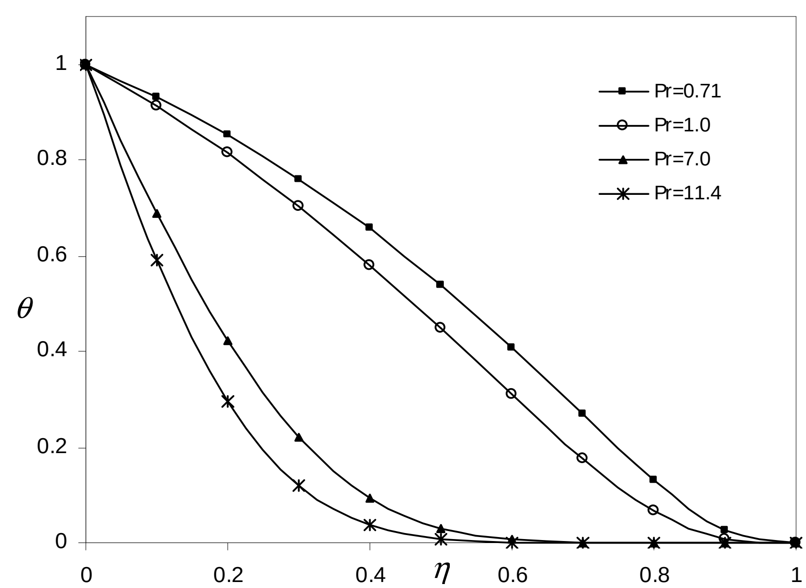

Fig. 9. Effect of Prandtl number Pr on temperature field *(Gr=5.0, Gm=5.0, So=1.0, NR=0.5, S=2.0, Sc=0.22, Kr=0.5, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)*

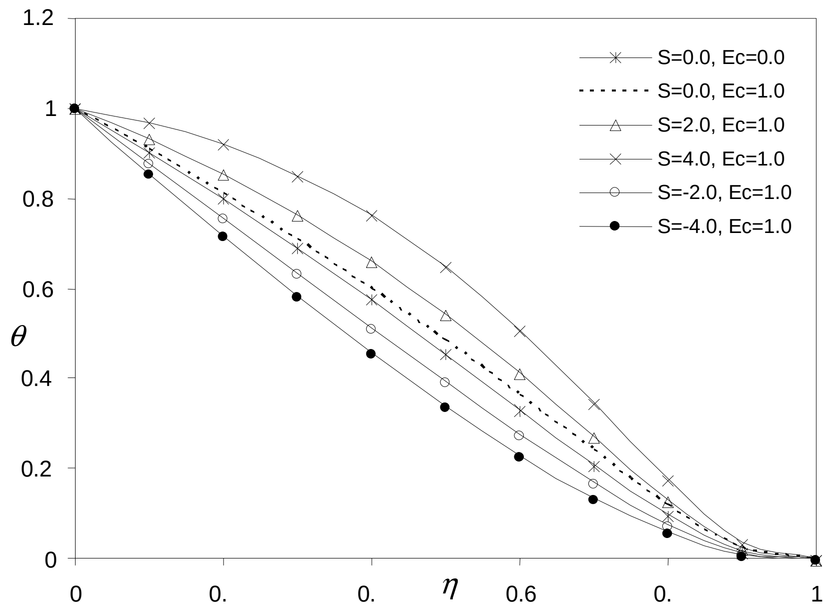

Fig. 10. Effect of of heat source/sink and viscous dissipation on temperature field
(Gr=5.0, Gm=5.0, So=1.0, NR=0.5, Pr=0.71, Sc=0.22, Kr=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

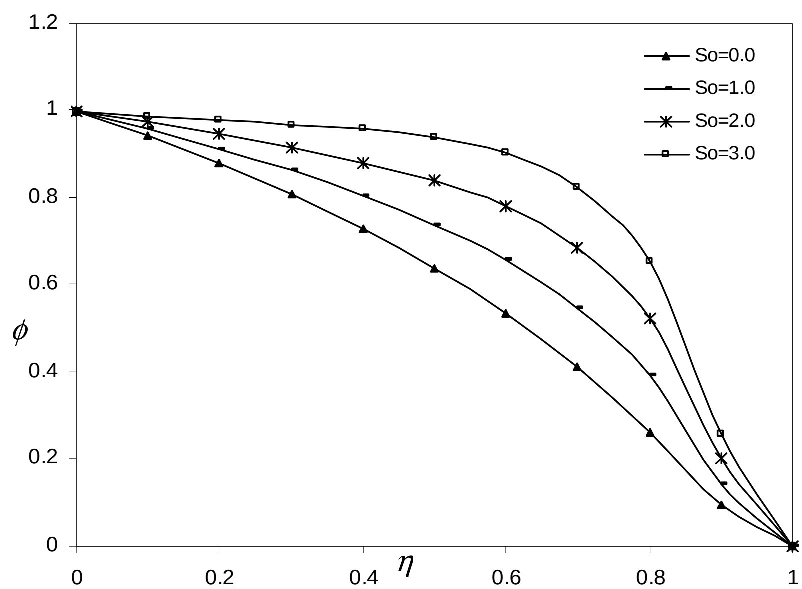

Fig. 11. Effect of Soret number So on Concentration field
(Gr=5.0, Gm=5.0, NR=0.5, Pr=0.71, S=2.0, Sc=0.22, Kr=0.5, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

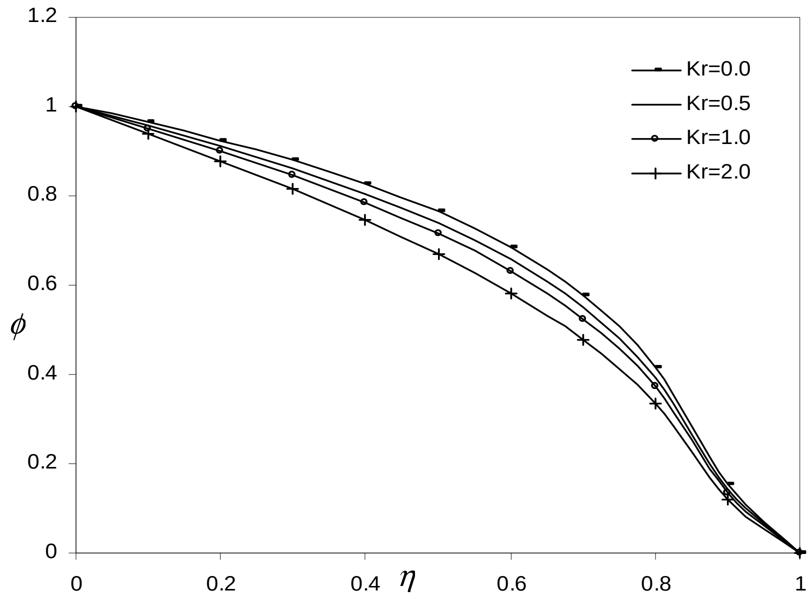

Fig. 12. Effect of chemical reaction parameter Kr on Concentration field
(Gr=5.0, Gm=5.0, NR=0.5, Pr=0.71, So=1.0, S=2.0, Sc=0.22, Kr=0.5, Ec=0.5, M=1.0, ε=0.01, n=0.1, A=0.3 and t=1.0)

The results obtained are compared with those of Srihari and Kesavareddy [28] for Skin-friction, rate of heat and mass transfer in the absence of heat source/sink parameter S and Eckert number Ec. The comparisons in all the cases are found to be in very good agreement.

5 Conclusions

Effects of Radiation and Soret number variation in the presence of heat source/sink on MHD unsteady laminar boundary layer flow of a chemically reacting incompressible viscous fluid along a semi-infinite vertical plate, is analysed. From this study the following conclusions are drawn.

- The temperature and velocity of the fluid increase in the presence heat source.
- Skin–friction and Nusselt number also increase in the presence of heat source and Eckert number. This due to the fact that effect of heat generation is to increase the rate of heat transport to the fluid thereby increasing the temperature and its velocity of the fluid
- Increasing values Eckert number is to increase the temperature of the fluid. This result agrees with fact that heat energy is stored in the fluid due to the frictional heating.
- An increase in So leads to increase in the velocity, but an increase in M leads to decrease in the velocity.
- The effect of heat source/sink on temperature is more significant than in the case of velocity field.
- Temperature and velocity increases as the radiation parameter increases. This due to the fact that an increase in the radiation parameter the rate of radiative heat transferred to

the fluid increases and consequently the fluid temperature and hence the velocity of its particles also increases

- The results obtained are compared with those of Srihari and Kesavareddy [28] for, Skin-friction, rate of heat and mass transfer in the absence of heat source/sink parameter S and Eckert number Ec. The comparisons in all the cases are found to be in very good agreement.

Competing Interests

Authors have declared that no competing interests exist.

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