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POWER TRANSMISSION OF A PARTICLE BEAM MOVING IN A RESISTIVE CYLINDRICAL TUBE

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ABSTRACT

The radial power transmission resulting from a particle beam of parabolic (quadratic) transverse charge distribution have been studied theoretically. The particle beam is moving at constant speed down a resistive cylindrical pipe of finite wall thickness. The wave equations for the electromagnetic fields induced by the beam motion inside the cylindrical pipe have been derived and solved. The coefficient of radial power transmission through the beam-pipe wall have been obtained analytically and then analyzed numerically for different beam energies, different wall conductivities and different wave mode frequencies. The radial power transmission is found to increase with increasing beam energy, to decrease with increasing wall conductivity and it is higher for the wave modes of lower frequencies.

Key Words: Power Transmission, Waveguides

I. INTRODUCTION

Most frequently, we encounter circumstances in which good shielding against electromagnetic fields is highly desired [1, 2]. For shielding based on reflection losses, two or more metallic layers separated by dielectric fillings lead to multiple reflections and provide more effective shielding than the same thickness of metal in single shield [3]. Multiple shield concepts are used in environment which require magmatic shielding in strong electromagnetic signals. The interaction of the magmatic fields of currents in each conductor due the electromotive force induced by the magnetic flux linkage around the conductors[4–6]. Consequently, like in the case of skin effect, the apparent A.C. resistance of the conductors is increased and the strength of the this shape (or proximity) effect will usually depend on the wave frequency, the gap width between the conductors and on their arrangement. The current unbalance due to the proximity effect in multiple shields can be reduced by spacing the conductors as far apart as possible. consequently, the skin effect becomes the predominant attenuation effect. In the opposite limit of conductors very close to each other, the apparent

A.C. resistance of the conductors is increased and shielding is predominantly due to the proximity effect[7–9]. In many cases, physical distance between the conductors will be enough to reduce their magnetic coupling to an acceptable levels. If two conductors are close to each other, their mutual inductance may perturb the current distribution and increase the effective resistance of the conductors. Reduction of the mutual inductance requires increasing the separation between conductors since the magnetic fields around the conductors are distance.

II. INDUCED ELECTROMAGNETIC FIELDS IN CYLINDRICAL TUBE

From Faradays and Amperes laws in a linear conducting medium, we have the following wave equations for the magnetic induction B and electric field E [10, 11]:

^{2}

$_{2} - \mu_0 E_0 \,\partial - \mu_0\, S\partial\, B(r, t) = -\mu_0 \times j_b(r, t)\,.$ (1)

$\partial t_2\, \partial t$

$\partial_2\, \partial\partial\ 1$

$_{2} - \mu_0 E_0 - \mu_0 S E(r, t) = \mu_0\, j_b(r, t) + \rho_b(r, t)\,.$ (2)

$\partial t_2\ \partial t\ \partial t E_0$where E_0 and μ_0 respectively, the free space permeability and S is the conductivity of medium under consideration. Here ρ_b and j_b are the beam charge and current densities, respectively. We consider a beam of particles of radius a with an axially symmetric transverse charge distribution

σ(r) which move at a constant speed along the axis of a cylindrical beam-pipe of radius b. With a longitudinal beam velocity such that v = βczˆalong the z axis, we have the following beam charge and current densities:
ρb(r, z, t)= σ(r)δ(z − βct) , (3) jb(r, t)= βcρb (r, t)ˆz. (4)

Here β is the relativistic factor and c is the speed of light in vacuum. For a uniformly charged disk of total charge Q.

$$Q = 2\pi\int_0^a \sigma(r) r dr. \tag{5}$$

The Fourier time-transformed beam charge and current densities are,

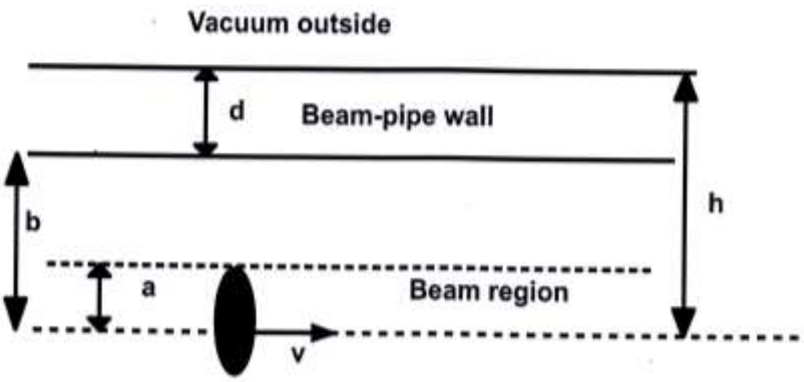

FIG. 1: Pipe geometry

$$\rho b(r,z,w) = \frac{\sigma(r)}{\beta c} e^{ik_z z} \tag{6}$$

$$jb(r,z,w) = \sigma(r) e^{ik_z z} \tag{7}$$

For a uniformly charged thin disk of radius charge Q, the surface charge density distribution in the transverse direction is

$$\sigma(r) = \frac{2Q}{\pi a^2}\left(1 - \frac{r^2}{a^2}\right) e^{ik_z z} \tag{8}$$

where k_z stands for number in the direction of beam propagation and $w = k_z \beta_c$ has been introduced.

For the axial symmetric beam of equations (3)and (4), only transverse magnetic (TM) modes couple to the propagating beam. the non-vanishing electro-magnetic field components are E_z, E_r and B_θ. The electromagnetic field components E_θ and B_r vanish identically because of the axial symmetry of the beam. Assuming a normal mode solution for electric E_z such that $\vec{E}$ (r, z, w) = $\vec{e}$ (r,w) $e^{ik_z z}$, and by making use of ρ (r, z, w) and $j_b(r, z,w)$ in equations (6) and (7), we obtain the following equations the longitudinal electric field component in each region of Fig 1 :

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{1d}{r\,dr} - \frac{k_z^2}{\gamma_0^2}\right]\vec{e}_z^{\,(1)}(r,w) = \frac{ik_z c}{\beta \in_0 c\gamma_0^2}\frac{2Q}{\pi a^2}\left(1\frac{r^2}{a^2}\right)0, \qquad a \le r \le a. \tag{9}$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{1d}{r\,dr} - \frac{k_z^2}{\gamma_0^2}\right]\vec{e}_z^{\,(2)}(r,w) = 0, \qquad a \le r \le b \tag{10}$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{1d}{r\,dr} - \frac{k^2}{\gamma_c^2}\right]\vec{e}_z^{\,(3)}(r,w) = 0, \qquad b \le r \le h = b + d. \tag{11}$$

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{1d}{r\,dr}-\frac{k^2}{\gamma_0^2}\right]\vec{e}_z^{(4)}(r,w)=0, \qquad h\le r<\infty \tag{12}$$

Here γ_0^{-2} has been introduced s follows:

$$\gamma_0^{-2}=1-\beta^2, \frac{1}{\gamma_0^2}=\frac{1}{\gamma_0^2}-\frac{iw\mu_0 S}{k_z^2} \tag{13}$$

The azimuthal magnetic field component h_θ (r, z, w) needed for matching the solutions at the different interfaces involved in the problem is obtained from Maxwell's curl equations as follows:

$$b_\theta=\frac{-i\gamma_c^2}{k_z}\frac{\beta}{c}\left(1\frac{S}{\in_0\, w}\right)\frac{\partial e_z}{\partial r} \tag{14}$$

Where γ in equation (14) stands for γ_0 in vacuum, for γ_c in the beam-pipe wall. The general solution for the z-component of the electric field is,

$$e_z(r,w)=\begin{cases} A_1 I_0(\sigma_0 r)-i\dfrac{2Q}{\pi a^2\in_0 k_z\beta c}\left(1-\dfrac{r^2}{a^2}-\dfrac{4}{\sigma_0^2a^2}\right) & r\le a\\ A_1 I_0(\sigma_0 r)+A_3K_0(\sigma_0 r) & a\le r\le b\\ A_1 I_0(\sigma_0 r)+A_3K_0(\sigma_c r) & b\le r\le h\\ A_1 I_0(\sigma_0 r) & h\le r<\infty \end{cases} \tag{15}$$

Where $\sigma_0=\dfrac{k_2}{\gamma_0}=\dfrac{w}{\beta_c\gamma_0},\sigma_c=\dfrac{k_2}{\gamma_c},I_0$ and k_0 are modified Bessel function of first and second kind respectively. The corresponding azimuthal magnetic field is,

$$b_\theta(r,w)=\begin{cases} -i\dfrac{\gamma_0^2}{k_z}\dfrac{\beta}{c}\left[A_1\sigma_0 I_1(\sigma_0 r)+i\alpha\dfrac{2}{a}\right] & 0<r<a\\ -i\dfrac{\gamma_0^2}{k_z}\dfrac{\beta}{c}\left[A_1\sigma_0 I_1(\sigma_0 r)-A_3\sigma_0 K_1(\sigma_0 r)\right] & a<r<b\\ -i\dfrac{\gamma_0^2}{k_z}\dfrac{\beta}{c}\left(1+i\dfrac{S}{w\in_0}\right)\left[A_1\sigma_0 I_1(\sigma_0 r)-A_3\sigma_c K_1(\sigma_c r)\right] & b<r<h\\ -i\dfrac{\gamma_0^2}{k_z}\dfrac{\beta}{c}\left[A_3\sigma_c K_1(\sigma_c r)\right] & h<r<\infty \end{cases} \tag{16}$$

Applying the boundary conditions on the tangential field components e_z and h_θ at all interfaces at $r = a$, $r = b$, $r = h$ we obtain the following closed system of algebraic equations for the integration constants:

$$A_1 I_0(\sigma_0 a) + ia\frac{4}{\sigma_0^2 a^2} = A_1 I_0(\sigma_0 a) + A_3 K_0(\sigma_0 a), \, continuity of\ E_z\ at\ r = a \tag{17}$$

$$A_1 I_1(\sigma_0 a) + ia\frac{2}{\sigma_0 a} = A_2 I_1(\sigma_0 a) + A_3 K_1(\sigma_c a), \, continuity of\ E_z\ at\ r = a \tag{18}$$

$$A_2 I_0(\sigma_0 b) + A_3 K_0(\sigma_0 b) = A_4 I_0(\sigma_c b) + A_5 K_0(\sigma_c b), \, continuity\ of\ E_z\ at\ r = b \tag{19}$$

$$\eta_c [A_2 I_0(\sigma_0 b) - A_3 K_1(\sigma_0 b) = A_4 I_0(\sigma_c b) + A_5 K_1(\sigma_c b), continuity\ of\ B_\theta\ at\ r = b \tag{20}$$

$$A_4 I_0(\sigma_0 h) + A_5 K_0(\sigma_c h) = A_6 K_0(\sigma_0 h), \, continuity\ of\ B_\theta\ at\ r = h \tag{21}$$

$$A_4 I_1(\sigma_c h) + A_5 K_0(\sigma_c h) = \eta_c A_6 K_1(\sigma_0 h), \, continuity\ of\ B_\theta\ at\ r = h \tag{22}$$

Where the parameters η_c and a are defined as follows

$$\eta_c = \frac{\gamma_0}{\gamma_c (1 + i\frac{\mu_0 c^2 S}{w})} = \frac{\gamma_0 w \in_0}{i\gamma_c (S - iw \in_0)}, \qquad a = \frac{2Q}{\pi a^2 w \in_0}$$

$$A_1 = 2i\alpha I_1(\sigma_1 a)\left[\frac{I_0(\sigma_1 a) + RK_0(\sigma_1 a)}{I_1(\sigma_1 a) - RK_1(\sigma_1 a)} - \frac{2}{(\sigma_1 a)}\right]\left[\frac{K_1(\sigma_1 a)}{I_1(\sigma_1 a)} - \frac{1}{R}\right], \tag{23}$$

$$A_2 = \frac{I_0(\sigma_1 a)}{I_1(\sigma_0 a) - RK_1(\sigma_0 a)} - A_1 + = \frac{ia\frac{2}{(\sigma_0 a)}}{I_1(\sigma_0 a) - RK_1(\sigma_0 a)} \tag{24}$$

$$A_3 = RA_2, \tag{25}$$

$$A_4 = \frac{I_0(\sigma_0 b) + RK_0(\sigma_1 b)}{I_0(\sigma_c b) - FK_0(\sigma_c b)} \tag{26}$$

$$A_5 = RA_4, \tag{27}$$

$$A_6 = \frac{I_0(\sigma_c h)}{K_0(\sigma_0 h)} A_4 + \frac{K_0(\sigma_c h)}{K_0(\sigma_0 h)} A_5, \tag{28}$$

$$R = \frac{\eta_c I_1(\sigma_0 b) - GI_0(\sigma_0 b)}{\eta_c K_1(\sigma_0 b) - GK_0(\sigma_0 b)} \tag{29}$$

$$G = \frac{I_1(\sigma_c b) - FK_1(\sigma_c b)}{I_0(\sigma_c b) + FK_0(\sigma_c b)} \tag{30}$$

$$F = \frac{I_1(\sigma_0 h) K_0(\sigma_0 h) + \eta_1 K_1(\sigma_0 h) I_0(\sigma_c h)}{K_1(\sigma_0 h) K_0(\sigma_0 h) - \eta_c K_1(\sigma_0 h) K_0(\sigma_c h)} \tag{31}$$

III. POWER TRANSMISSION COEFFICIENT

We define the beam-pipe transmission coefficient of longitudinal electric field τ_z as the ratio of the longitudinal electric field leaking into the outer vacuum at $r = h$ to the field impinging on the pipe-wall at $r = b$, namely,

$$\tau_z = \frac{E_z(r=h)}{E_z(r=b)} = \frac{I_0(\sigma_c h) + \frac{A_5}{A_4} K_0(\sigma_0 h)}{I_0(\sigma_c b) + \frac{A_5}{A_4} K_0(\sigma_0 b)}$$
$$= \frac{I_0(\sigma_c h) + FK_0 \frac{A_5}{A_4}(\sigma_c h)}{I_0(\sigma_c b) + FK_0 K_0(\sigma_c b)} \qquad (32)$$

Where F is given by the following expression

$$F \approx \frac{1}{\pi} e^{2\sigma_c h} \left[\frac{K_0(\sigma_0 h) + \eta_c K_1(\sigma_0 h)}{K_0(\sigma_0 h) - \eta_c K_1(\sigma_0 h)} \right] \qquad (33)$$

Introduce u such that:

$$u = \left[\frac{K_0(\sigma_0 h) + \eta_c K_1(\sigma_0 h)}{K_0(\sigma_0 h) - \eta_c K_1(\sigma_0 h)} \right] \qquad (34)$$

We obtain

$$\tau_z = \sqrt{\frac{b}{h}} \varepsilon^{\sigma_c d} \frac{1 + F\pi\, e^{-2\sigma_c d}}{1 + F\pi\, e^{-2\sigma_c d}} = \sqrt{\frac{b}{h}} \varepsilon^{\sigma_c d} \frac{1+u}{1+ue^{2\sigma_c d}}$$
$$= \sqrt{\frac{b}{b+d}} \frac{1+u}{e^{-2\sigma_c d} + ue^{\sigma_c d}} \qquad (35)$$

Substitute for u and rearrange, we get,

$$\tau_z = \sqrt{\frac{b}{h}} \frac{K_0(\sigma_0 h) - \eta_c(\sigma_0 h) + k_0(\sigma_0 h) + \eta_c k_1(\sigma_0 h)}{(K_0(\sigma_0 h) - \eta_c K_1(\sigma_0 h) e^{-\sigma_e d} + (K_0(\sigma_0 h) + \eta_c K_1(\sigma_0 h))e^{\sigma_e d}}$$
$$= \sqrt{\frac{b}{h}} \frac{2K_0(\sigma_0 h)}{2K_0(\sigma_0 h)\cosh(\sigma_0 h) - \eta_c K_1(\sigma_0 h)(e^{-\sigma_e d} - e^{\sigma_e d})}$$
$$= \sqrt{\frac{b}{h}} \frac{1}{\cosh(\sigma_c h) - \eta_c \frac{K_1(\sigma_0 h)}{K_1(\sigma_0 h)} \sinh(\sigma_c d)} \qquad (36)$$

In a similar way, the transmission of the radial electric field (or azimuthal magnetic field) is found to be,

$$\tau_z = \frac{E_r(r=h)}{E_r(r=b)} = \frac{I_0(\sigma_c h) - \frac{A_5}{A_4} K_1(\sigma_0 h)}{I_1(\sigma_c b) - \frac{A_5}{A_4} K_1(\sigma_c h)} \qquad (37)$$

$$\tau_z = \sqrt{\frac{b}{h}} \frac{1-u}{e^{-\sigma_c d} - ue^{-\sigma_c d}} \qquad (38)$$

$$\tau_z = \sqrt{\frac{b}{h}}\,\frac{K_0(\sigma_0 h) - \eta_c(\sigma_0 h) + k_0(\sigma_0 h) + \eta_c k_1(\sigma_0 h)}{(K_0(\sigma_0 h) - \eta_c K_1(\sigma_0 h)e^{-\sigma_e d} + (K_0(\sigma_0 h) + \eta_c K_1(\sigma_0 h))e^{\sigma_e d}}$$

$$= \sqrt{\frac{b}{h}}\,\frac{2\eta_c K_1(\sigma_0 h)}{-2\sinh(\,\sigma_c h)K_0(\sigma_0 h) + 2\eta_c \cosh(\,\sigma_c d)K_1(\sigma_0 h)}$$

$$= \sqrt{\frac{b}{h}}\,\frac{2\eta_c K_1(\sigma_0 h)}{2K_0(\sigma_0 h)\sinh(\,\sigma_c h)K_0(\sigma_0 h) + \cosh(\,\sigma_0 h)K_1(\sigma_0 h)} \tag{39}$$

We can now obtain the coefficient of the radial power transmission τ_p. It is defined as the ratio of time averaged radial power leaving the wall at $r = h$ to the radial power entering the wall at $r = b$, namely

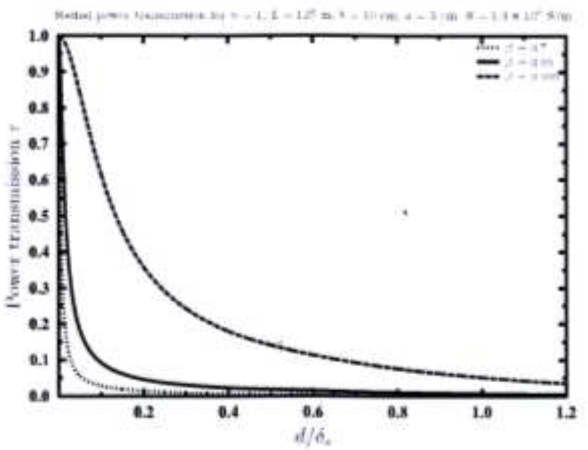

FIG. 2: The radial power transmission for different beam speeds at the harmonic number n = 1 and the wall conductivity S = 1.4 × 10^6.

$$\tau_z = \frac{S_r^{(p)}(r=h)}{S_r^{(p)}(r=b)} = \frac{E_z(r=h)H_0^*(r=h)}{E_z(r=b)H_0^*(r=b)} = \tau_z \tau_0^* \tag{40}$$

$$\tau_z = \frac{E_z(r=h)}{E_z(r=b)}\sqrt{\frac{b}{h}}\,\frac{1}{\cosh(\,\sigma_c d) - \eta_c \sinh(\,\sigma_c d)K_1(\sigma_c h)K_1(\sigma_0 h)}, \tag{41}$$

$$\tau_r = \frac{H_\theta(r=h)}{H_\theta(r=b)} \approx \sqrt{\frac{b}{h}}\,\frac{\eta_c\, K_1(\sigma_0 h)/K_0(\sigma_0 h)}{\eta_c \cosh(\sigma_0 d)K_1(\sigma_0 h)/K_0(\sigma_0 h) - \sinh(\,\sigma_c d)} \tag{42}$$

where $S_r^{(p)}$ is the radial component of the time averaged Poynting's vector

IV. NUMERICAL EXAMPLE

We obtained analytically the coefficient of the radial power transmission τ_p. It has been defined as the ratio of time averaged radial power leaving the wall at h to the radial power entering the wall at $r = b$, namely,

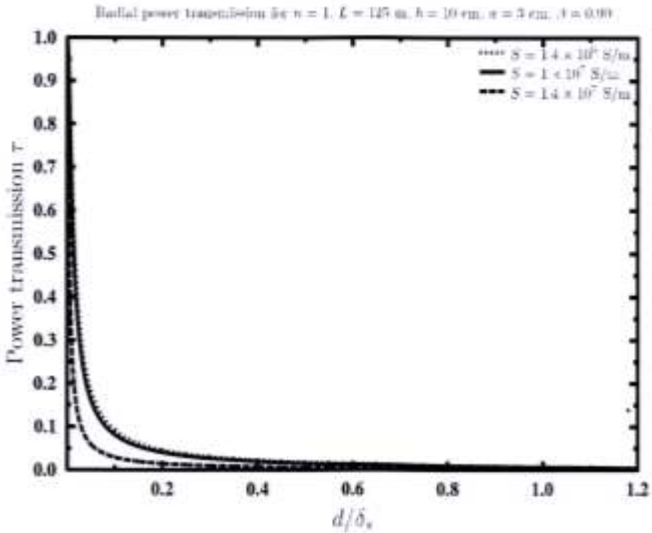

FIG. 3: The radial power transmission for different wall conductivities at the harmonic number at $n = 1$ and the beam speed $\beta = 0.99$.

$$\tau_p = \frac{S_r^{(p)}(r=h)}{S_r^{(p)}(r=b)} = \frac{E_z(r=h)H_\theta^*(r=h)}{E_z(r=b)H_\theta^*(r=b)} = \tau_z \tau_0^* \tag{43}$$

$$\tau_r = \frac{H_z(r=h)}{H_z(r=b)} \approx \sqrt{\frac{b}{h}} \frac{1}{\cosh(\sigma_c d) - \eta_c \sinh\ K_1(\sigma_0 h)/K_1(\sigma_0 h)/K_0(\sigma_c h)} \tag{44}$$

$$\tau_r = \frac{H_\theta(r=h)}{H_\theta(r=b)} \approx \sqrt{\frac{b}{h}} \frac{\eta_c\ K_1(\sigma_0 h)/K_0(\sigma_0 h)}{\eta_c \cosh(\sigma_0 d) K_1(\sigma_0 h)/K_0(\sigma_0 h) - \sinh(\ \sigma_c d)} \tag{45}$$

Where $S_r^{(p)}$ is the radial component of the time averaged Poynting's vector _S= 12 _E × _H_. We summarize our numerical results as follows,

1. Figure 2 show the power transmission at the lowest harmonic numbern = 1 for the beam energies *b* = 0.7, *b* = 0.9 and *b* = 0.99. We see that the power transmission becomes an important issue at high energies due to increases in the power transmission with increasing beam energy.
2. The curves of Figure 3 show that the power transmission will decrease by increase conductivity.
3. In figure Figure 4 we see that the radial power transmission is higher for the lower harmonic numbers. Also in accelerator design, one should take care of the lower harmonics.

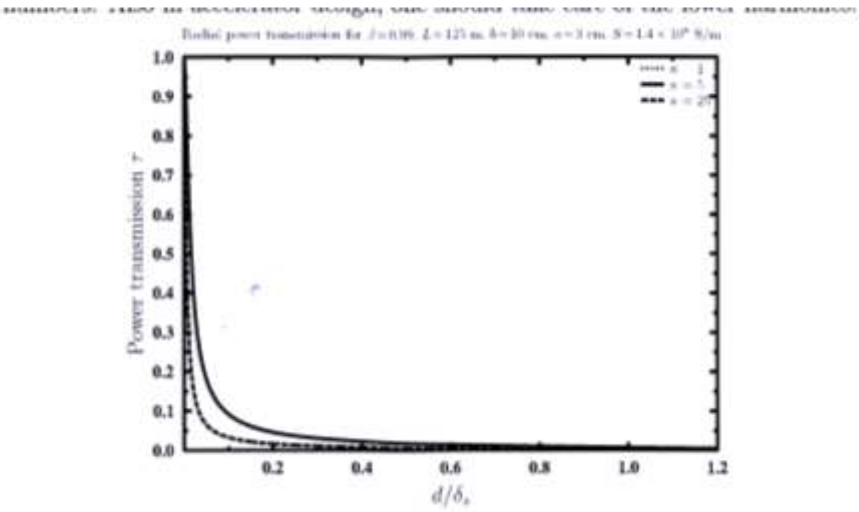

FIG. 4: The radial power transmission for different harmonic numbers at the beam speed $\beta = 0.99$ and wall conductivity $S = 1.4 \times 10^6$.

V. CONCLUSIONS

Power radial transmission of a particle beam of parabolic (quadratic) transverse charge distribution has been presented in this paper, theoretically and numerically. The beam is moving at constant speed down a resistive cylindrical beam-pipe of finite wall thickness. The coefficient of radial power transmission through the beam-pipe wall have been obtained analytically and then analyzed numerically for different beam energies, different wall conductivities and different wave mode (harmonic) frequencies. The radial transmission of power is a measure of the shielding effectiveness of the pipe wall.

We obtained analytically the coefficient of the radial power transmission _p. It has been defined as the ratio of time averaged radial power leaving the wall at h to the radial power entering the wall at r = b, namely,

$$\tau_p = \frac{S_r(r=h)}{S_r(r=b)} = \frac{E_z(r=h)H_\theta^*(r=h)}{E_z(r=b)H_\theta^*(r=b)} = \tau_z \tau_0^* \tag{46}$$

$$\tau_r = \frac{H_z(r=h)}{H_z(r=b)} \approx \sqrt{\frac{b}{h}} \frac{1}{\cosh(\sigma_c d) - \eta_c \sinh\ K_1(\sigma_0 h)/K_1(\sigma_0 h)/K_0(\sigma_c h)} \tag{47}$$

$$\tau_r = \frac{H_\theta(r=h)}{H_\theta(r=b)} \approx \sqrt{\frac{b}{h}} \frac{\eta_c\ K_1(\sigma_0 h)/K_0(\sigma_0 h)}{\eta_c \cosh(\sigma_0 d)K_1(\sigma_0 h)/K_0(\sigma_0 h) - \sinh(\ \sigma_c d)} \tag{48}$$

where S_r is the radial component of the time averaged Poynting's vector $\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}*$. We now summarize the main results and conclusions of the analytical and numerical calculations as follows:

1. the radial power transmission increases with increasing beam energy (see Fig 2),
2. the radial power transmission decreases with increasing wall conductivity (see Fig 3),
3. the radial power transmission is higher for lower harmonic numbers (see Fig 4),

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